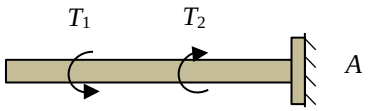
3-1. A shaft has an outer diameter of 36 mm and an inner diameter of 26 mm. Two torques $T_1$ and $T_2$ are applied to the shaft as shown. If $T_1 \sim N\left(70, 7^2\right)$ Nm and $T_2 \sim N\left(30, 2^2\right)$ Nm, determine the distribution of the maximum shear stress developed in the shaft. Assuming that $T_1$ and $T_2$ are independent.




**Solution:**

The maximum shear stress in the shaft could be obtianed by

$$\tau_{\max} = \frac{T_{\max} c}{J} = \frac{(T_1 - T_2)c}{J} = \frac{(T_1 - T_2)(0.036/2)}{\frac{\pi}{2}(0.018^4 - 0.013^4)} = 0.15(T_1 - T_2)$$

Since $T_1$ and $T_2$ are independent, $\tau_{\max}$ also follows a normal distribution. We have

$$\mu_{\tau_{\max}} = 0.15(\mu_{T_1} - \mu_{T_2}) = 0.15(70 - 30) = 6 \text{ MPa}$$

$$\sigma_{\tau_{\max}} = 0.15\sqrt{\sigma_{T_1}^2 + \sigma_{T_2}^2} = 0.15\sqrt{7^2 + 2^2} = 1.09 \text{ MPa}$$

Thus, $\tau_{\max}$ follows a normal distribution $\tau_{\max} \sim N\left(6, 1.09^2\right)$ MPa. **Ans.**