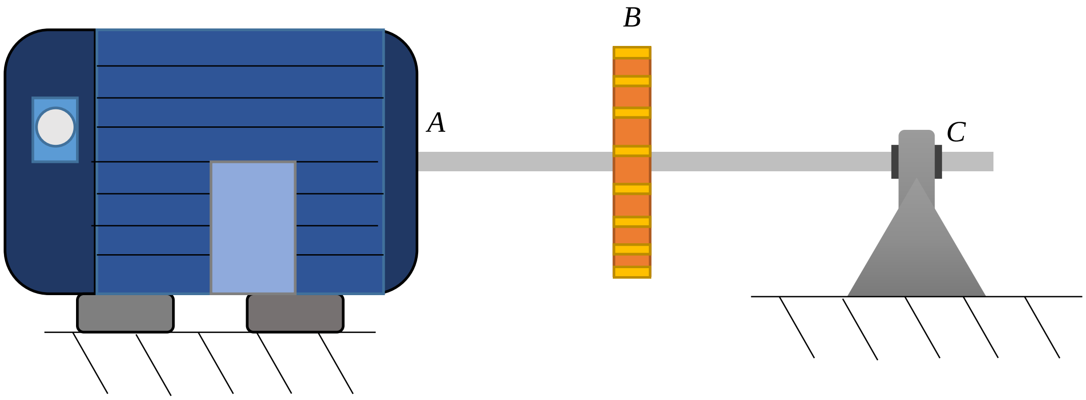
3-10. A shaft is supported by a smooth bearing at C and has an outer diameter $D = 30$ mm and a thickness $t = 5$ mm. The motor operates at an angular velocity $\omega = 50$ rad/s and delivers $P \sim N\left(5, 0.5^2\right)$ kW of power. What is the probability the shaft will fail if the allowable torsion shear stress is $\tau_a \sim N\left(35, 4^2\right)$? Assume P and τ_a are independent.



Solution:

Torque:

$$T = \frac{P}{\omega}$$

$$\mu_T = \frac{\mu_P}{\omega} = \frac{5000}{50} = 100 \text{ N·m} \quad \textbf{(1)}$$

$$\sigma_T = \frac{\sigma_T}{\omega} = \frac{500}{50} = 10 \text{ N·m} \quad \textbf{(2)}$$

$T \sim N\left(100, 10^2\right)\left(10^3\right)$ N·mm due to unit balance.

Torsion shear stress: The polar moment of inertia is $J = \frac{\pi}{32}\left(D^4 - d^4\right) = 63813.60 \text{ mm}^4$.

$$\tau = \frac{Tc}{J}$$

$$\mu_\tau = \frac{\mu_T\left(\frac{D}{2}\right)}{J} = 23.51 \text{ MPa} \quad \textbf{(3)}$$

$$\sigma_\tau = \frac{\sigma_T\left(\frac{d}{2}\right)}{J} = 2.35 \text{ MPa} \tag{4}$$

$$\tau \sim N\left(23.51, 2.35^2\right) \text{ MPa}$$

Probability of failure:

$$p_f = \Pr(\tau > \tau_a) = \Pr(Y = \tau_a - \tau < 0) \tag{5}$$

Since $P \sim N\left(5, 0.5^2\right)$ kW, and $\tau_a \sim N\left(35, 4^2\right)$ MPa are independent, Y also follows a normal distribution of $Y \sim N(\mu_y, \sigma_y^2)$.

$$\mu_y = \mu_{\tau_a} - \mu_\tau = 35 - 23.51 = 11.49 \text{ MPa} \tag{6}$$

$$\sigma_y = \sqrt{\sigma_{\tau_a}^2 + \sigma_\tau^2} = \sqrt{4^2 + 2.35^2} = 4.64 \text{ MPa} \tag{7}$$

Equation (5) can be written as

$$p_f = \Pr\left(Y < 0\right) = \Pr\left(\frac{Y - \mu_y}{\sigma_y} < \frac{-\mu_y}{\sigma_y}\right) = \Phi\left(\frac{-\mu_y}{\sigma_y}\right) = \Phi\left(-2.4774\right) = 6.6170\left(10^{-3}\right)$$ **Ans.**