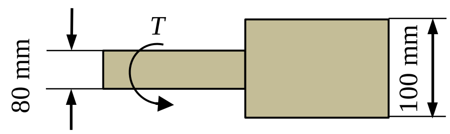
3-2. The shaft rotates with $\omega = 600\,\text{rpm}$ and transmits the power of $P \sim N(25, 3^2)\,\text{kW}$. The allowable shear stress of the shaft is $\tau_a \sim N(15, 2^2)\,\text{MPa}$. Determinate the probability of failure of the shaft. Given that the torsional stress-concentration factor is $K = 1.3$. Assuming that P and τ_a are independent.



Solution:

$$\omega = 600(2\pi)/60 = 20\pi(\text{rad/s})$$

The torque on the shaft is

$$T = \frac{P}{\omega} = \frac{P}{20\pi}$$

The shear stress developed in the shaft is

$$\tau = K\frac{Tc}{J} = 1.3 \times \frac{\frac{P}{20\pi}(0.04)}{\frac{\pi}{2}(0.04)^4} = 0.21 \times 10^3 P$$

Set $Y = \tau_a - \tau$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{\tau_a} - \mu_\tau = \mu_{\tau_a} - 0.21 \times 10^3 \mu_P = 15 \times 10^6 - 0.21 \times 10^3 \times 25 \times 10^3 = 9.75\,\text{MPa}$$

$$\sigma_Y = \sqrt{\sigma_{\tau_a}^2 + \sigma_\tau^2} = \sqrt{\sigma_{\tau_a}^2 + (0.21 \times 10^3)^2 \sigma_P^2} = \sqrt{(2)^2 + (0.63)^2} = 2.1\,\text{MPa}$$

Thus, the probability of failure of the shaft is

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-4.64) = 1.74 \times 10^{-6}$$ **Ans.**