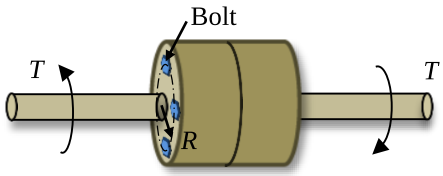
3-3. A coupling with 4 bolts connects the two shafts as shown. Assume that the shear stress in the bolts is uniform. The torque T applied on the shafts follows a normal distribution $T \sim N\left(100,\ 10^2\right)\ \mathrm{N \cdot m}$. Each bolt has a diameter $d = 0.01\,\mathrm{m}$, and the bolts are uniformly distributed at the radius $R = 0.02\,\mathrm{m}$. If the allowable shear stress of the bolt is $\tau_a \sim N\left(20,\ 1.5^2\right)\mathrm{MPa}$, determine the probability of failure the bolt. Assume that T and τ_a are independent.



Solution:

$$T - nFR = 0\,; \qquad F = \frac{T}{nR}$$

in which n is the number of bolts, and F is the shear force in each bolt.

The shear stress developed in the bolt is

$$\tau = \frac{F}{A} = \frac{T}{nRA} = \frac{T}{nR\left(\dfrac{\pi d^2}{4}\right)} = \frac{4T}{4(0.02)(3.14)(0.01)^2} = 1.59 \times 10^5 T$$

Set $Y = \tau_a - \tau$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{\tau_a} - \mu_\tau = \mu_{\tau_a} - 1.59 \times 10^5 \mu_T = 24 \times 10^6 - 1.59 \times 10^5 \times 100 = 8.1\,\mathrm{MPa}$$

$$\sigma_Y = \sqrt{\sigma_{\tau_a}^2 + \sigma_\tau^2} = \sqrt{\sigma_{\tau_a}^2 + (1.59 \times 10^5)^2 \sigma_T^2} = \sqrt{(1.5)^2 + (1.59)^2} = 2.19\,\mathrm{MPa}$$

Thus, the probability of failure of the bolt is

$$p_f = \Pr\,(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-3.7\right) = 1.08 \times 10^{-4} \qquad \textbf{Ans.}$$