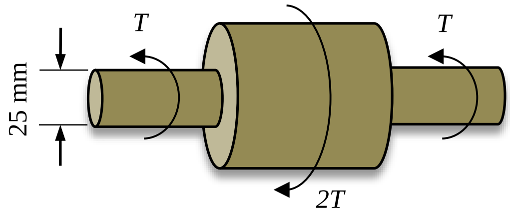
3-5. A step shaft has an allowable shear stress of $\tau_a \sim N(14, 0.8^2)$ MPa . The torque on the shaft follows $T \sim N(22, 1.8^2)$ N·m . Determine the probability of failure of the shaft. Given that the torsional stress-concentration factor is $K = 1.3$. Assume that T and τ_a are independent.



Solution:

The maximum shear stress developed in the shaft is

$$\tau_{\max} = K\frac{Tc}{J} = 1.3 \times \frac{T(0.025/2)}{\frac{\pi}{2}\left(\frac{0.025}{2}\right)^4} = 0.42 \times 10^6 T$$

Set $Y = \tau_a - \tau_{\max}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{\tau_a} - \mu_{\tau_{\max}} = 14 \times 10^6 - 0.42 \times 10^6 \mu_T = 14 \times 10^6 - 0.42 \times 10^6 \times 22 = 4.76 \times 10^6 \text{ Pa}$$

$$\sigma_Y = \sqrt{\sigma_{\tau_a}^2 + \sigma_{\tau_{\max}}^2} = \sqrt{(0.8 \times 10^6)^2 + (0.42 \times 10^6)^2 \sigma_T^2} = 1.23 \times 10^6 \text{ Pa}$$

Thus, the probability of failure of the shaft could be obtained by

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-3.87) = 5.44 \times 10^{-5}$$ **Ans.**