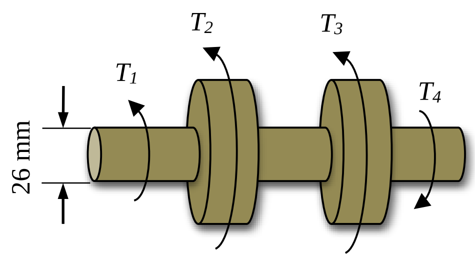
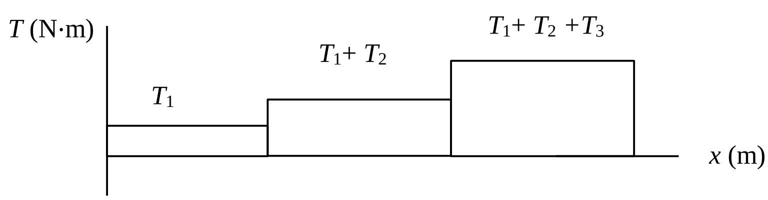
3-6. A shaft transmits torques applied to the gears. The three torques follow normal distributions $T_1 \sim N(120, 10^2)\ \mathrm{N \cdot m}$, $T_2 \sim N(250, 20^2)\ \mathrm{N \cdot m}$, and $T_3 \sim N(160, 15^2)\ \mathrm{N \cdot m}$, respectively. Determine the distribution of the maximum shear stress on the shaft. Assume that T_1, T_2 and T_3 are independent.



Solution:

The intrrnal torque is shown on the torque diagram.



From the torque diagram, we have $T_{\max} = T_1 + T_2 + T_3$. Then, applying torsion Formula

$$\tau_{\max} = \frac{T_{\max} c}{J} = \frac{(0.026/2)}{\frac{\pi}{2}\left(\frac{0.026}{2}\right)^4}\left(T_1 + T_2 + T_3\right) = 0.29 \times 10^6 \left(T_1 + T_2 + T_3\right)$$

Thus, $\tau_{\max}$ also follows a normal distribution. We have

$$\mu_{\tau_{\max}} = 0.29 \times 10^6 \mu_T = 0.29 \times 10^6 \left(\mu_{T_1} + \mu_{T_2} + \mu_{T_3}\right) = 0.29 \times 10^6 \left(120 + 250 + 160\right) = 153.7\mathrm{MPa}$$

$$\sigma_{\tau_{\max}} = 0.29 \times 10^6 \sigma_T = 0.29 \times 10^6 \sqrt{10^2 + 20^2 + 15^2} = 7.8\ \mathrm{MPa}$$

Thus, $\tau_{\max}$ follows a normal distribution $\tau_{\max} \sim N\left(153.7,\ 7.8^2\right)\ \mathrm{MPa}$. **Ans.**