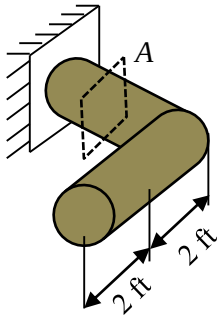
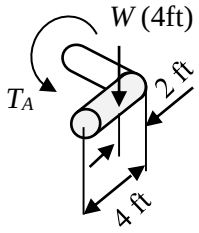
3-7. A rod has a diameter of 1.2 in, and its weight follows a random distribution $W \sim N\left(12,\, 0.8^2\right)$ lb/ft. Determine the distribution of the maximum torsional stress in the rod at a section lacated at *A* due to the rod's weight. Assume that only the the internal torque is taken into account.



Solution:

Since we are only interested in the internal torque, from the free-body diagram, we have



$$\Sigma M_x = 0; \qquad T_A - W(4)(2) = 0 \qquad T_A = 8W(\frac{12\text{in}}{1\text{ft}}) = 96W$$

The polar moment of inertia of the cross section at *A* is $J = \frac{\pi}{2}\left(\frac{1.2}{2}\right)^4 = 0.203\,\text{in}^4$. We have

$$\tau_{\max} = \frac{T_A C}{J} = \frac{96\,(0.6)W}{0.203} = 283.09\,W$$

Thus, $\tau_{\max}$ also follows a normal distribution, and

$$\mu_{\tau_{\max}} = 283.09\mu_W = 283.09\left(12\right) = 3397\text{ psi}$$

$$\sigma_{\tau_{\max}} = 283.09\sigma_W = 283.09(0.8) = 226.5\text{ psi}$$

Thus, $\tau_{\max}$ follows a normal distribution $\tau_{\max} \sim N\left(3397,\, 226.5^2\right)$ MPa. **Ans.**