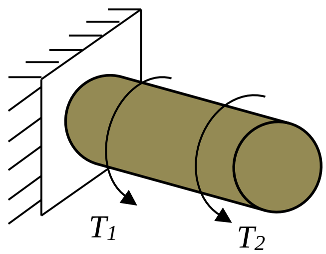
3-8. A rod has a diameter of 36 mm, and it is subjected to two torques, $T_1 \sim N\left(400,\, 40^2\right)\,\mathrm{N\cdot m}$ and $T_2 \sim N\left(500,\, 30^2\right)\,\mathrm{N\cdot m}$. The allowable torsional stress of this rod is $\tau_a \sim N(145, 12^2)$ MPa. Determine the probability of failure of this rod. Assume that $\tau_a$, $T_1$ and $T_2$ are independent.




**Solution:**

The maximum shear stress in the rod is

$$\tau_{\max} = \frac{T_{\max} c}{J} = \frac{(T_1 + T_2)c}{J} = \frac{(T_1 + T_2)(0.036/2)}{\frac{\pi}{2}(0.018^4)} = 0.11 \times 10^6 (T_1 + T_2)$$

Set $Y = \tau_a - \tau_{\max}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{\tau_a} - \mu_{\tau_{\max}} = \mu_{\tau_a} - 0.11 \times 10^6 (\mu_{T_1} + \mu_{T_2}) = 145 - 0.11 \times 10^6 (400 + 500) = 46\ \mathrm{MPa}$$

$$\sigma_Y = \sqrt{\sigma_{\tau_a}^2 + \sigma_{\tau_{\max}}^2} = \sqrt{\sigma_{\tau_a}^2 + \left(0.11 \times 10^6\right)^2 \left(\sigma_{T_1}^2 + \sigma_{T_2}^2\right)} = \sqrt{12^2 + \left(0.11 \times 10^6\right)^2 \left(40^2 + 30^2\right)} = 13.2\ \mathrm{MPa}$$

Thus, the probability of failure of the rod could be obtained by

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-3.48) = 2.5 \times 10^{-4}$$ **Ans.**