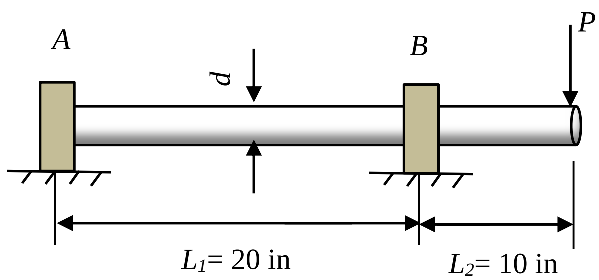
4-1. A vertical random force $P \sim N(600, 50^2)$ lb acts on a shaft as shown in the following figure. The sleeve bearing at A and B support only vertical forces. The diameter of the shaft is $d = 1.5$ in and the allowable bending stress of the shaft is $S_a \sim N(26, 3^2)$ ksi. Determine the probability of failure of the shaft.



Solution

The free-body diagram of this shaft is shown in Fig. 1. The shear and moment diagrams are shown in Fig. 2.

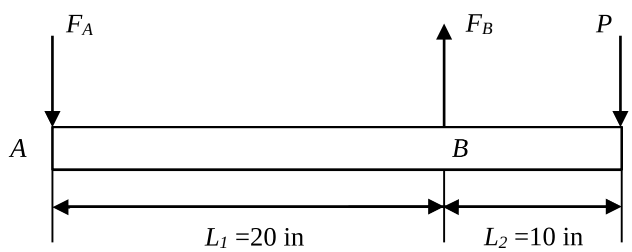


Fig. 1

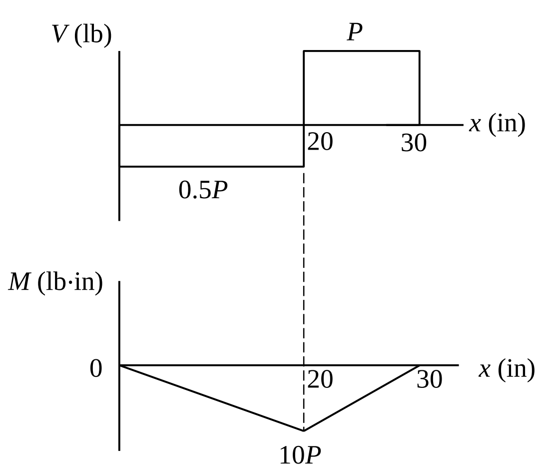


Fig. 2

$\circlearrowleft +\Sigma M_A = 0$, $F_B(20) - P(30) = 0$, $F_B = 1.5P$

$\circlearrowleft +\Sigma M_B = 0$, $F_A(20) - P(10) = 0$, $F_A = 0.5P$

As indicated in the moment diagram, $M_{\max} = 10P$.

The moment of inertia of the cross section about the neutral axis is $I = \dfrac{\pi}{64} d^4$.

Since $c = \dfrac{d}{2}$, $S_{\max} = \dfrac{M_{\max} c}{I} = \dfrac{(10P)\left(\frac{d}{2}\right)}{\left(\frac{\pi}{64} d^4\right)} = \dfrac{320P}{\pi d^3} = \dfrac{320P}{3.14(1.5)^3} = 30.2P$.

Set $Y = S_a - S_{\max}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{S_a} - \mu_{S_{\max}} = \mu_{S_a} - 30.2\mu_P = 26 \times 10^3 - 30.2(600) = 7880 \text{ psi}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + (30.2)^2 \sigma_P^2} = \sqrt{(3000)^2 + (30.2)^2 (50^2)} = 3358.59 \text{ psi}$$

The probability of failure of the shaft is then given by

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-7880}{3358.59}\right) = \Phi(-2.346) = 0.0095.$$ **Ans.**