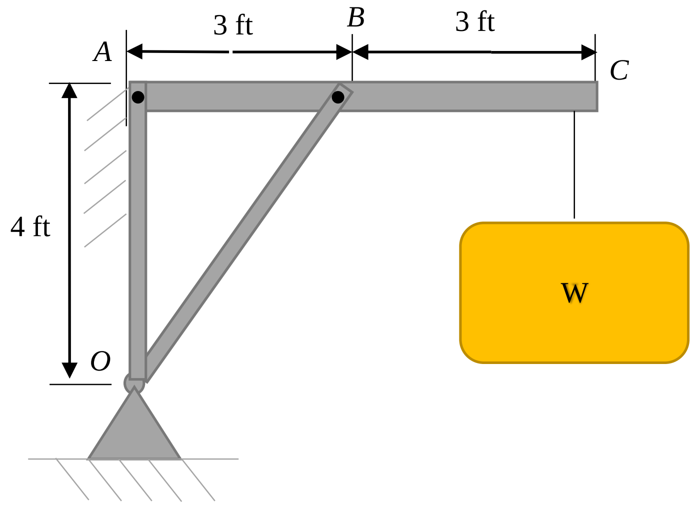
4-8. The system is used to support a normally distributed weight $W \sim N\left(1200, 100^2\right)$ lb. If the yield stress of rod *AC* is $S_y \sim N\left(1500, 200^2\right)$ psi, what is the probability of failure? Consider only the normal stress at *A*. The diameter of rod *AC* is d = 2 in. Assume W and S_y are independent.



Solution

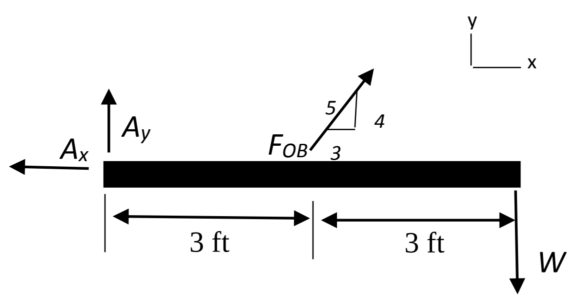
Sum the forces: The force at A_x can be found by summing the forces in the *x* and *y* directions and then take the moments about *A*.

$$+\uparrow \sum F_y = 0; \quad -W + F_{OB}(4/5) + A_y = 0;$$

$$+\rightarrow \sum F_x = 0; \quad -A_x + F_{OB}(3/5) = 0;$$

$$+\nwarrow \sum M_A = 0; \quad F_{OB}(4/5)(3) - W(6) = 0;$$

$$F_{OB} = 2.5W; \quad A_x = 1.5W; \quad A_y = -W;$$



For the probability of failure at A:

$$p_f = \Pr(S_{Ax} > S_y) = \Pr(Y = S_y - S_{Ax} < 0) = \Pr\left(Y = S_y - \frac{A_x}{A} < 0\right) = \Pr\left(Y = S_y - \frac{1.5W}{A} < 0\right) \quad \textbf{(1)}$$

Since $W \sim \mathrm{N}\left(1200, 100^2\right)$ lb, $S_y \sim N\left(1500, 200^2\right)$ psi, and *W* and S_y are independent, *Y* also follows a normal distribution. $Y \sim N(\mu_y, \sigma_y^2)$.

$$\mu_y = \mu_{s_y} - \left(\frac{1.5}{A}\right)\mu_w = 1500 - \left(\frac{1.5}{\frac{\pi 2^2}{4}}\right)1200 = 927.04 \text{ psi} \tag{2}$$

$$\sigma_y = \sqrt{\sigma_{S_y}^2 + \left(\frac{1.5}{A}\right)^2 \sigma_W^2} = \sqrt{200^2 + \left(\frac{1.5}{\frac{\pi 2^2}{4}}\right)^2 (100)^2} = 205.62 \text{ psi} \tag{3}$$

Equation (1) can be written as

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_y}{\sigma_y} < \frac{-\mu_y}{\sigma_y}\right) = \Phi\left(\frac{-\mu_y}{\sigma_y}\right) = \Phi(-4.51) = 3.24\left(10^{-6}\right) \qquad \textbf{Ans.}$$