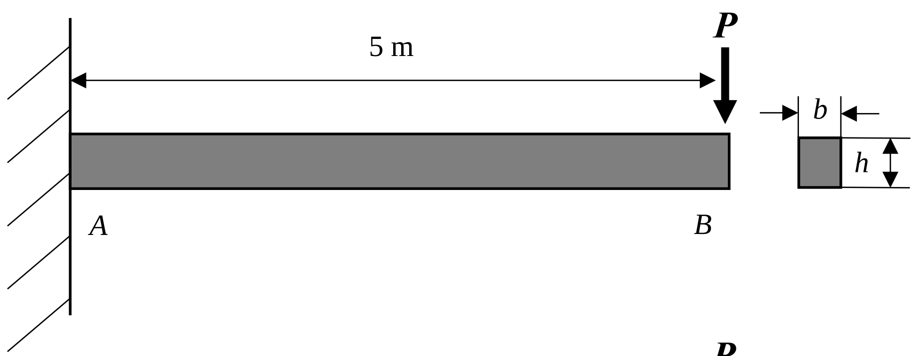
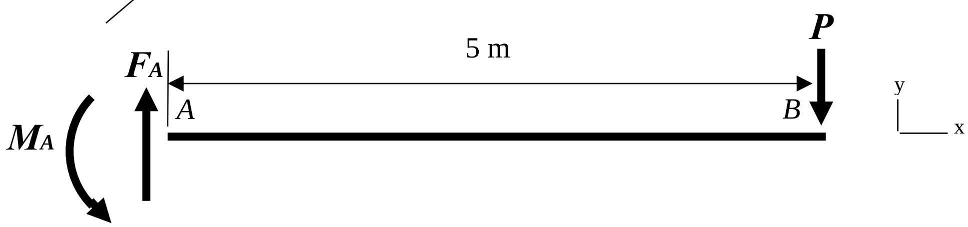
4-9. A beam is subjected to a normally distributed force $P \sim N\left(2, 0.2^2\right)$ kN as shown. If the maximum allowable bending stress is $S_a \sim N\left(35, 3.5^2\right)$ MPa, what is the probability of failure? The beam is rectangular with $h = 150$ mm and $b = 100$ mm. Assume P and S_a are independent.



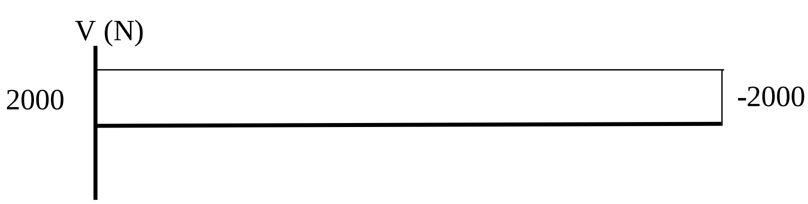


Solution:

Sum the forces: To find the largest bending moment, sum the forces in the y direction and take the moment about A. The

$+\uparrow \sum F_y = 0;$ $\quad -F_y - P = 0;$ $\quad F_y = -P$

$+\circlearrowleft \sum M_A = 0;$ $\quad M_A - (5000)P = 0;$ $\quad M_A = (5000)P$



By plugging in the average of P, the maximum moment is at point *A* with magnitude

$$M_{\max} = (5000)P$$

Moment of inertia: Calculate the moment of inertia about the *z* axis.

$$I = \frac{bh^3}{12} = \frac{(100)(150)^3}{12} = 2.8125\left(10^8\right)\ \text{mm}^4$$

Bending stress: Calculate the distribution of stress that is developed in the beam.

$$\sigma_{\max} = \frac{Mc}{I};$$

$$\mu_{\sigma_{\max}} = \frac{c}{I}(5000)\mu_P = \left(\frac{\frac{150}{2}}{\left(2.8125\left(10^8\right)\right)}\right)(5000)(2000) = 26.67\ \text{MPa} \quad \textbf{(1)}$$

$$\sigma_{\sigma_{\max}} = \frac{c}{I}(5000)\sigma_P = \left(\frac{\frac{150}{2}}{\left(2.8125\left(10^8\right)\right)}\right)(5000)(200) = 2.667\ \text{MPa} \quad \textbf{(2)}$$

$$\sigma_{\max} \sim N\left(26.67, 2.667^2\right)\ \text{MPa}$$

Probability of failure:

$$p_f = \Pr(\sigma_{\max} > S_a) = \Pr(Y = S_a - \sigma_{\max} < 0) \quad \textbf{(3)}$$

Since $P \sim N\left(2, 0.2^2\right)$ kN and $S_a \sim N\left(35, 3.5^2\right)$ MPa are independent, Y also follows a normal distribution. $Y \sim N(\mu_Y, \sigma_Y^2)$.

$$\mu_Y = \mu_{S_a} - \mu_{\sigma_{\max}} = 35 - 26.67 = 8.33 \text{ MPa} \tag{4}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \sigma_{\sigma_{\max}}^2} = \sqrt{3.5^2 + 2.667^2} = 4.40 \text{ MPa} \tag{5}$$

Equation (3) can be written as

$$p_f = \Pr\left(Y < 0\right) = \Pr\left(\frac{Y - \mu_y}{\sigma_y} < \frac{-\mu_y}{\sigma_y}\right) = \Phi\left(\frac{-\mu_y}{\sigma_y}\right) = \Phi\left(-1.8939\right) = 0.0291$$ **Ans.**