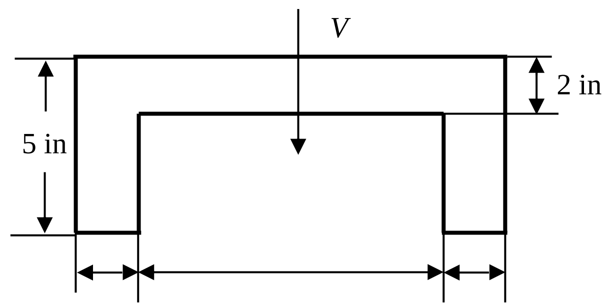
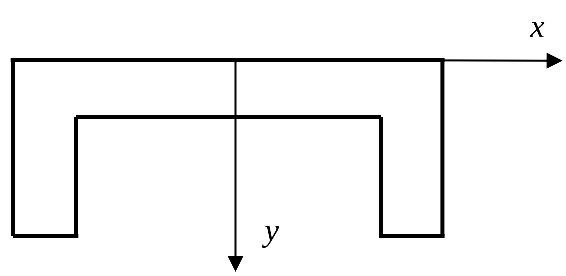
5-1. A shear force $V \sim N\left(20, 1.8^2\right)$ kip is applied to a member with the cross section as shown. If the allowable shear stress is $\tau_a \sim N\left(4, 0.5^2\right)$ ksi, determine the probability of failure of the member. Assume that τ_a and V are independent.



Solution:



The cordinate system is shown in the figure, then

$$\bar{y} = \frac{\sum_{i=1}^{3} y_i A_i}{\sum_{i=1}^{3} A_i} = \frac{(1)\left[(2)(8)\right]+(3.5)\left[(2)(3)(2)\right]}{(2)(8)+(2)(3)(2)} = 2.0714\,\text{in}$$

Using the Parallel Axis Theorem, the moment of inertia about the centroidal axis is

$$I = \frac{1}{12}(8)(2^3)+(8)(2)(2.0714-1)^2+(2)\left(\frac{1}{12}\right)(2)(3^3)+(2)(2)(3)(3.5-2.0714)^2 = 57.2\,\text{in}^3$$

Thus, the first moment of the cross section area at the neutral axis is

$$Q_{\max} = \Sigma \overline{y}' A' = (2)\left[\frac{1}{2}(5-2.0714)\right]\left[(2)(5-2.0714)\right] = 17.153\,\text{in}^3$$

The maximum shear stress is

$$\tau_{\max} = \frac{VQ_{\max}}{I\,t} = \frac{17.153}{57.2(2)(2)}V = 0.075V$$

Set $Y = \tau_a - \tau_{\max}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{\tau_a} - \mu_{\tau_{\max}} = \mu_{\tau_a} - 0.075\mu_V = 4 - 0.075(20) = 2.5\,\text{ksi}$$

$$\sigma_Y = \sqrt{\sigma_{\tau_a}^2 + \sigma_{\tau_{\max}}^2} = \sqrt{\sigma_{\tau_a}^2 + (0.075)^2\sigma_V^2} = \sqrt{(0.5)^2 + (0.135)^2} = 0.517\,\text{ksi}$$

Thus, the probability of failure of the member is

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-4.836) = 6.64\times 10^{-7}$$ **Ans.**