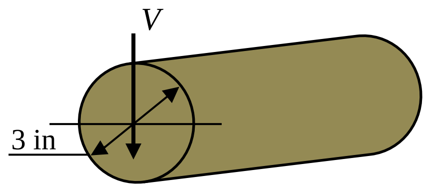
5-2. A rod has a diameter of 3 in, and it is subjected to a shear force of $V \sim N\left(6,\, 0.5^2\right)$ kip. If the allowable shear stress is $\tau_a \sim N\left(3,\, 0.5^2\right)$ ksi, determine the probability of failure of the rod. Assume that τ_a and V are independent.



Solution:

$$\overline{y}' = \frac{4r}{3\pi} = \frac{4(1.5)}{3\pi} = 0.637 \text{ in}$$

$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}\pi(1.5)^4 = 3.974 \text{ in}^3$$

The first moment of the cross section area at the neutral axis is

$$Q_{\max} = \overline{y}'A' = 0.637\left[\frac{\pi}{2}(1.5)^2\right] = 2.25 \text{ in}^3$$

The maximum shear stress is

$$\tau_{\max} = \frac{VQ_{\max}}{I\,t} = \frac{2.25}{3.974(3)}V = 0.1887V$$

Set $Y = \tau_a - \tau_{\max}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{\tau_a} - \mu_{\tau_{\max}} = \mu_{\tau_a} - 0.1887\mu_V = 3 - 0.1887(6) = 1.868 \text{ ksi}$$

$$\sigma_Y = \sqrt{\sigma_{\tau_a}^2 + \sigma_{\tau_{\max}}^2} = \sqrt{\sigma_{\tau_a}^2 + (0.1887)^2\sigma_V^2} = \sqrt{(0.5)^2 + (0.094)^2} = 0.509 \text{ksi}$$

Thus, the probability of failure of the rod is

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-3.67\right) = 1.21\times 10^{-4}$$ **Ans.**