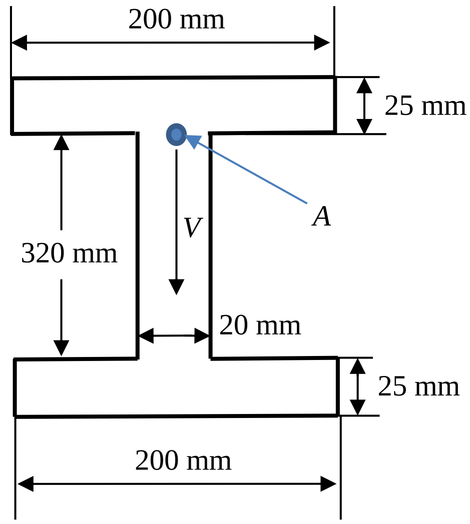
5-3. A shear force $V \sim N\left(22, 2^2\right)$ kN is applied to a wide-flange beam with the cross section as shown. If the allowable shear stress on the web at A is $\tau_a \sim N\left(5, 0.5^2\right)$ MPa, determine the probability of failure of the beam. Assume that τ_a and V are independent.



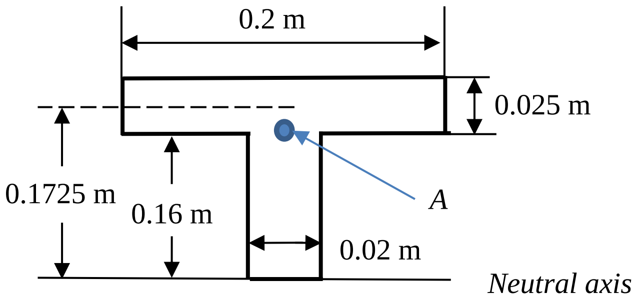
Solution:

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.37^3) - \frac{1}{12}(0.18)(0.32^3) = 3.527 \times 10^{-4}\ \text{m}^3$$

From the following figure, we have

$$Q_A = \overline{y}'A' = 0.1725(0.2)(0.025) = 8.625 \times 10^{-4}\ \text{m}^3$$



Applying the shear formula,

$$\tau_A = \frac{VQ_A}{I\,t} = \frac{8.625\times10^{-4}}{(3.527\times10^{-4})(0.02)}V = 122.27V$$

Set $Y = \tau_a - \tau_{\max}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{\tau_a} - \mu_{\tau_A} = \mu_{\tau_a} - 122.27\mu_V = 5\times10^6 - 122.27(22\times10^3) = 2.31\,\text{MPa}$$

$$\sigma_Y = \sqrt{\sigma_{\tau_a}^2 + \sigma_{\tau_A}^2} = \sqrt{\sigma_{\tau_a}^2 + (122.27)^2\sigma_V^2} = \sqrt{(0.5\times10^6)^2 + (122.27)^2(2\times10^3)^2} = 0.557\,\text{MPa}$$

Thus, the probability of failure of the beam is

$$p_f = \Pr(Y<0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-4.147) = 1.68\times10^{-5}$$ **Ans.**