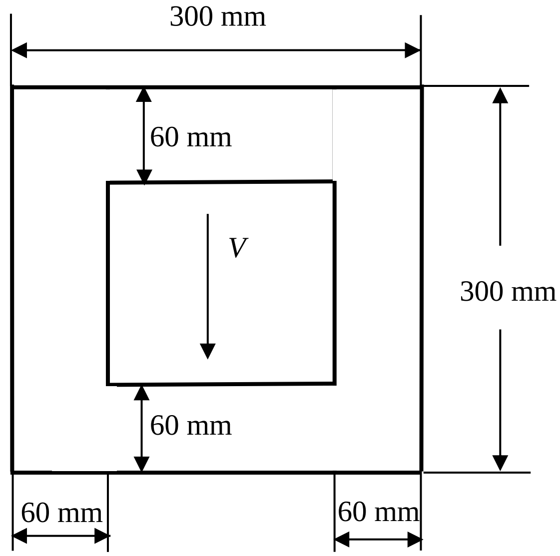
5-4. A shear force $V \sim N\left(120, 20^2\right)\text{kN}$ is applied to a square beam with the cross section as shown. If the allowable shear stress is $\tau_a \sim N\left(8, 0.6^2\right)\text{MPa}$, determine the probability of failure of the beam. Assume that τ_a and V are independent.



Solution:

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.3)(0.3^3) - \frac{1}{12}(0.18)(0.18^3) = 5.875\times10^{-4}\ \text{m}^3$$

Then, we have

$$Q_{\max} = \Sigma\overline{y}'A' = 0.12(0.18)(0.06) + 2(0.075)(0.15)(0.06) = 0.001296 + 0.00135 = 26.46\times10^{-4}\,\text{m}^3$$

Applying the shear formula,

$$\tau_{\max} = \frac{VQ_{\max}}{I\,t} = \frac{26.46\times10^{-4}}{(5.875\times10^{-4})(0.12)}V = 37.53V$$

Set $Y = \tau_a - \tau_{\max}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{\tau_a} - \mu_{\tau_{\max}} = \mu_{\tau_a} - 37.53\mu_V = 8\times10^6 - 37.53(120\times10^3) = 3.496\ \text{MPa}$$

$$\sigma_Y = \sqrt{\sigma_{\tau_a}^2 + \sigma_{\tau_{\max}}^2} = \sqrt{\sigma_{\tau_a}^2 + (37.53)^2\sigma_V^2} = \sqrt{(0.6\times10^6)^2 + (37.53)^2(20\times10^3)^2} = 0.96\ \text{MPa}$$

Thus, the probability of failure of the beam is

$$p_f = \Pr(Y<0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-3.638) = 1.37\times10^{-4}$$ **Ans.**