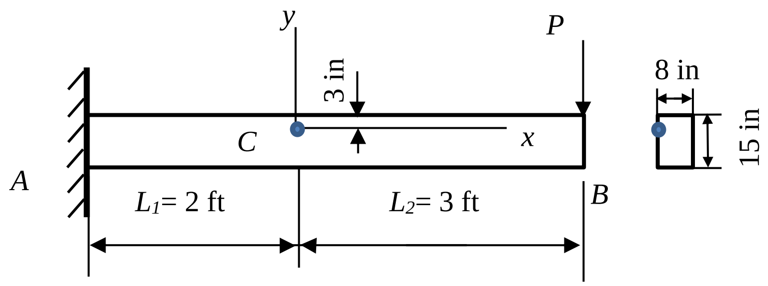
6-10. A vertical random force P is acting on the beam as shown in the figure. The strain in the x direction at point C is measured and follows a normal distribution $\varepsilon_x \sim N(80\times10^{-6}, (8\times10^{-6})^2)$. Determine the distribution of the force P . Given that the beam has a Young's modulus of $E = 29\times10^3$ ksi . P and ε_x are independent.



Solution:

Section Properties

$$I = \frac{1}{12}(8)(15^3) = 2250\,\text{in}^4 \tag{1}$$

$$Q_C = \overline{y}'A' = 6(8)(3) = 144\,\text{in}^3 \tag{2}$$

Normal Stress

$$\sigma = E\varepsilon_x = \left(29\times10^3\right)\varepsilon_x \tag{3}$$

$$\sigma = \frac{My}{I} = \frac{3(P)(12)(4.5)}{2250} = 0.072P \tag{4}$$

Thus, from equation (3) and (4), we have

$$\left(29\times10^3\right)\varepsilon_x = 0.072P \tag{5}$$

$$P = \left(4.03\times10^5\right)\varepsilon_x \tag{6}$$

Since $\varepsilon_x \sim N(80\times10^{-6}, (8\times10^{-6})^2)$, we have

$$\mu_P = \left(4.03\times10^5\right)\mu_{\varepsilon_x} = 32.24\,\text{kip} \tag{7}$$

$$\sigma_P = \left(4.03\times10^5\right)\sigma_{\varepsilon_x} = 3.22\text{ kip} \tag{8}$$

Thus, force P follows $P \sim N(32.24, 3.22^2)$ kip . **Ans.**