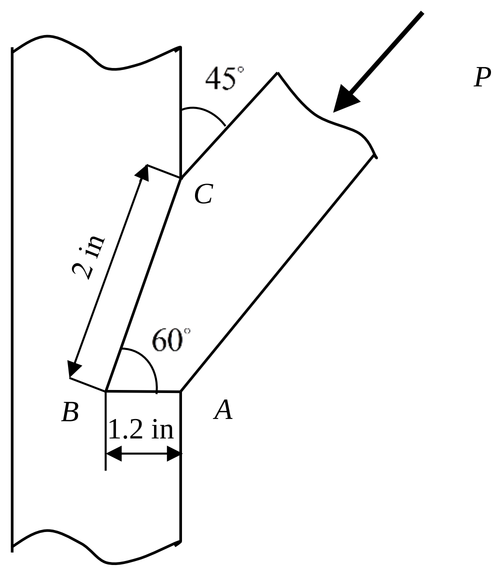
6-11. An axial member force P is applied to the joint of a component as shown in the figure. Assume that P follows a normal distribution $P \sim N\left(2, 0.2^2\right)$ kip, and the allowable normal stress on the joint section follows $S_a \sim N\left(2.5, 0.2^2\right)$ ksi. If the member is 1.2-in thick, determine the probabilities of failure of section AB and BC. Assume P and S_a are independent.



Solution:

$$\sum F_x = 0, \quad N_{BC}\cos 30^\circ - P\sin 45^\circ = 0$$

$$\sum F_y = 0, \quad N_{AB} - P\cos 45^\circ - N_{BC}\sin 30^\circ = 0$$

Solve the above equations, we have

$$N_{AB} = \left(\frac{\sqrt{6}}{6} + \frac{\sqrt{2}}{2}\right)P$$

$$N_{BC} = \frac{\sqrt{6}}{3}P$$

Thus, the normal stresses on section AB and BC are

$$S_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{\left(\frac{\sqrt{6}}{6} + \frac{\sqrt{2}}{2}\right)P}{(1.2)(1.2)} = 0.775P$$

$$S_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{\frac{\sqrt{6}}{3}P}{(2)(1.2)} = 0.34P$$

Set $Y_1 = S_a - S_{AB}$, then $Y_1 \sim N(\mu_{Y_1}, \sigma_{Y_1}^2)$, where

$$\mu_{Y_1} = \mu_{S_a} - \mu_{S_{AB}} = 0.95\,\text{ksi}$$

$$\sigma_{Y_1} = \sqrt{\sigma_{S_a}^2 + \sigma_{S_{AB}}^2} = 0.25\,\text{ksi}$$

The probability of failure on section *AB* is

$$p_{f1} = \Pr(Y_1 < 0) = \Phi\left(\frac{-\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi(-3.7588) = 8.5356 \times 10^{-5}$$ **Ans.**

Set $Y_2 = S_a - S_{BC}$, then $Y_2 \sim N(\mu_{Y_2}, \sigma_{Y_2}^2)$, where

$$\mu_{Y_2} = \mu_{S_a} - \mu_{S_{BC}} = 1.82\,\text{ksi}$$

$$\sigma_{Y_2} = \sqrt{\sigma_{S_a}^2 + \sigma_{S_{BC}}^2} = 0.21\,\text{ksi}$$

The probability of failure on section *BC* is

$$p_{f2} = \Pr(Y_2 < 0) = \Phi\left(\frac{-\mu_{Y_2}}{\sigma_{Y_2}}\right) = \Phi(-8.6131) = 3.1761 \times 10^{-13}$$ **Ans.**