6-12. When a car brakes its rear wheels, the tires may slip on the road. The normal force on each rear tire is F_n, and the coefficient of kinetic friction between the tires and the road is $\upsilon_k = 0.45$. Assume the rubber of the tires is flexible and the air pressure of each tire follows $P \sim N\left(30,\, 3^2\right)$ psi . If the allowable shear stress of the tire follows $S_a \sim N\left(22,\, 2^2\right)$ psi , determine the probability of failure. Assume that *P* and S_a are independent.

Solution:

The friction force between the tire and the road

$$F = F_n\left(\upsilon_k\right) = 0.45F_n$$

Set A as the contact area between each tire and the road, then, using $P = \frac{F_n}{A}$, we have

$$A = \frac{F_n}{P}$$

The average shear stress is given by

$$S = \frac{F}{A} = \frac{0.45F_n}{A} = 0.45P$$

Set $Y = S_a - S$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{S_a} - \mu_S = \mu_{S_a} - 0.45\mu_P = 22 - 0.45(32) = 8.5\,\text{kip}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \sigma_S^2} = \sqrt{\sigma_{S_a}^2 + (0.45)^2\,\mu_P^2} = 2.413\,\text{kip}$$

Thus, the probability of failure is

$$p_f = \Pr\left(Y < 0\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-3.5226\right) = 2.1366\times10^{-4}$$ **Ans.**