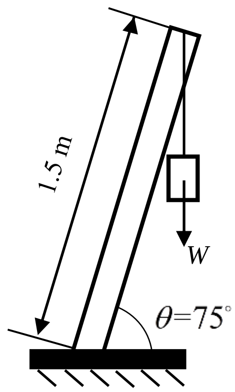
6-4. A pole with a fixed bottom is used to support a weight $W \sim N(800, 60^2)$ N as shown in the figure. The pole has a radius of 18 mm. The mass of the pole is negligible. If the allowable tensile stress of the pole follows $S_a \sim N(110, 10^2)$ MPa, determine the probability of failure. Assume that W and S_a are independent.



Solution:

Section Properties

$$A = \pi(0.018)^2 = 0.001\,\text{m}^2$$

$$I = \frac{1}{4}\pi(0.018^4) = 8.2448 \times 10^{-8}\ \text{m}^4$$

The maximum tensile stress is

$$S_T = \frac{-W\sin\theta}{A} + \frac{Mc}{I} = \frac{-W\sin 75^\circ}{A} - \frac{W(1.5)\cos 75^\circ(0.015)}{I} = \left(8.3809\times10^4\right)W$$

Since $W \sim N(800, 60^2)$ N, we have

$$\mu_{S_T} = \left(8.3809\times10^4\right)\mu_W = 67.047\ \text{MPa}$$

$$\sigma_{S_T} = \left(8.3809\times10^4\right)\sigma_W = 5.0285\,\text{MPa}$$

Set $Y = S_a - S_T$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{S_a} - \mu_{S_T} = 110 - 67.047 = 42.953\,\text{MPa}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \sigma_{S_T}^2} = \sqrt{10^2 + (5.0285)^2} = 11.193\,\text{MPa}$$

Thus, the probability of failure of the pole is obtained by

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-3.8374\right) = 6.2164\times10^{-5} \qquad \textbf{Ans.}$$