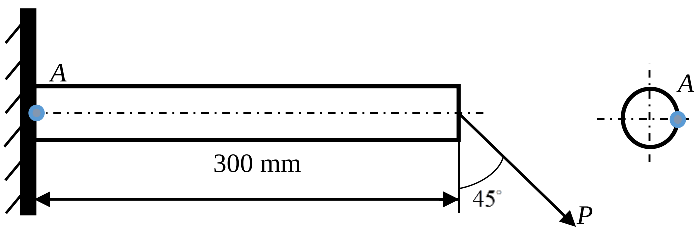
6-6. A random force $P \sim N(1, 0.5^2)$ kN is acting on a bar, which has a radius of 16 mm as shown in the figure. Determine the probabilities of failure at point *A* located at the left end due to excessive normal stress and shear stress. The allowable normal stress follows $S_{a1} \sim N(3, 0.2^2)$ MPa, and the allowable shear stress follows $S_{a2} \sim N(3.5, 0.3^2)$ MPa. Assume that P, S_{a1} and S_{a2} are independent.



Solution:

Section Properties

$$A = \pi(0.016)^2 = 8.04 \times 10^{-4}\ \text{m}^2$$

$$I = \frac{1}{4}\pi(0.016^4) = 5.15 \times 10^{-8}\ \text{m}^4$$

$$Q_A = \overline{y}'A' = \left(\frac{4(0.016)}{3\pi}\right)\left(\frac{\pi(0.016)^2}{2}\right) = 2.73 \times 10^{-6}\,\text{m}^4$$

Normal Stress at Point *A*

$$S_1 = \frac{P\sin 45^\circ}{A} + \frac{Mc}{I} = \frac{P\sin 45^\circ}{A} + 0 = (879.22)\,P$$

Since $P \sim N(1, 0.5^2)$ kN, we have

$$\mu_{S_1} = (879.22)\,\mu_P = 0.88\,\text{MPa}$$

$$\sigma_{S_1} = (879.22)\sigma_P = 0.44\,\text{MPa}$$

Set $Y_1 = S_{a1} - S_1$, then $Y_1 \sim N(\mu_{Y_1}, \sigma_{Y_1}^2)$, where

$$\mu_{Y_1} = \mu_{S_{a1}} - \mu_{S_1} = 2.12\,\text{MPa}$$

$$\sigma_{Y_1} = \sqrt{\sigma_{S_{a1}}^2 + \sigma_{S_1}^2} = 0.48\,\text{MPa}$$

Thus, the probability of failure due to excessive normal stress is

$$p_{f1} = \Pr(Y_1 < 0) = \Pr\left(\frac{Y_1 - \mu_{Y_1}}{\sigma_{Y_1}} < \frac{-\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi\left(\frac{-\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi(-4.3912) = 5.6368\times 10^{-6}$$ **Ans.**

Shear Stress at Point A

$$S_2 = \frac{VQ_A}{It} = \frac{(P\cos 45^\circ)Q_A}{It} = (1.17\times 10^3)\,P$$

Thus, we have

$$\mu_{S_2} = (1.17\times 10^3)\,\mu_P = 1.17\,\text{MPa}$$

$$\sigma_{S_2} = (1.17\times 10^3)\,\sigma_P = 0.59\,\text{MPa}$$

Set $Y_2 = S_{a2} - S_2$, then $Y_2 \sim N(\mu_{Y_2}, \sigma_{Y_2}^2)$, where

$$\mu_{Y_2} = \mu_{S_{a2}} - \mu_{S_2} = 2.33\,\text{MPa}$$

$$\sigma_{Y_2} = \sqrt{\sigma_{S_{a2}}^2 + \sigma_{S_2}^2} = 0.66\,\text{MPa}$$

Thus, the probability of failure due to excessive shear stress is

$$p_{f2} = \Pr(Y_2 < 0) = \Pr\left(\frac{Y_2 - \mu_{Y_2}}{\sigma_{Y_2}} < \frac{-\mu_{Y_2}}{\sigma_{Y_2}}\right) = \Phi\left(\frac{-\mu_{Y_2}}{\sigma_{Y_2}}\right) = \Phi(-3.5351) = 2.038\times 10^{-4}$$ **Ans.**