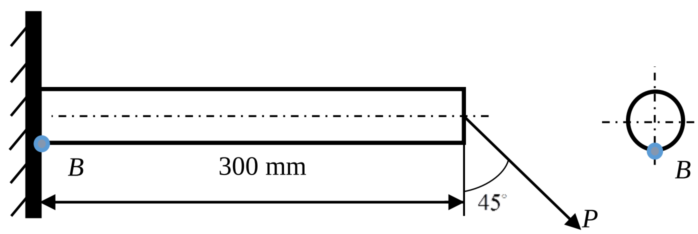
6-7. A random force $P \sim N(700, 50^2)$ N is acting on a bar, which has a radius of 45 mm as shown in the figure. Determine the probability of failure at point *B* located at the left end due to excessive normal stres. The allowable normal stress follows $S_{a1} \sim N(3, 0.2^2)$ MPa . Assume that P , and S_{a1} are independent.



Solution:

Section Properties

$$A = \pi(0.045)^2 = 0.0064\,\text{m}^2$$

$$I = \frac{1}{4}\pi(0.045^4) = 3.22\times10^{-6}\ \text{m}^4$$

$$Q_B = 0$$

Normal Stress at Point *B*

$$S_1 = \frac{P\sin 45^\circ}{A} - \frac{Mc}{I} = \frac{P\sin 45^\circ}{A} - \frac{P\cos 45^\circ(0.3)(0.045)}{I} = (-2852.9)\,P$$

Since $P \sim N(700, 50^2)$ N , we have

$$\mu_{S_1} = (-2852.9)\,\mu_P = -1.997\,\text{MPa} \quad \text{(Negative value means compressive stress)}$$

$$\sigma_{S_1} = (-2852.9)\,\sigma_P = -0.143\,\text{MPa} \quad \text{(Negative value means compressive stress)}$$

Set $Y_1 = S_{a1} - S_1$, then $Y_1 \sim N(\mu_{Y_1}, \sigma_{Y_1}^2)$, where

$$\mu_{Y_1} = \mu_{S_{a1}} - \mu_{S_1} = 1.003\ \text{MPa}$$

$$\sigma_{Y_1} = \sqrt{\sigma_{S_{a1}}^2 + \sigma_{S_1}^2} = 0.246\ \text{MPa}$$

Thus, the probability of failure due to excessive normal stress is

$$p_{f1} = \Pr(Y_1 < 0) = \Pr\left(\frac{Y_1 - \mu_{Y_1}}{\sigma_{Y_1}} < \frac{-\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi\left(\frac{-\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi(-4.083) = 2.2234 \times 10^{-5}$$ **Ans.**