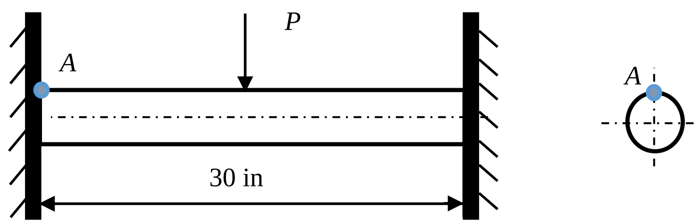
6-8. A random force $P \sim N(200, 20^2)$ lb is acting on the center of a shaft that has a radius of 0.5 in. Determine the probability of failure at point A due to excessive normal stress. The allowable normal stress follows $S_{a1} \sim N(30, 3^2)$ ksi. Assume that P, and S_{a1} are independent.



Solution:

Section Properties

$$A = \pi(0.5)^2 = 0.7854\,\text{in}^2$$

$$I = \frac{1}{4}\pi(0.5^4) = 0.0491\,\text{in}^4$$

Normal Stress at Point A

$$S_1 = \frac{Mc}{I} = \frac{P(30/2)}{I} = (76.39)\,P$$

Since $P \sim N(200, 20^2)$ lb, we have

$$\mu_{S_1} = (76.39)\,\mu_P = 15.28\,\text{ksi}$$

$$\sigma_{S_1} = (76.39)\,\sigma_P = 1.53\,\text{ksi}$$

Set $Y_1 = S_{a1} - S_1$, then $Y_1 \sim N(\mu_{Y_1}, \sigma_{Y_1}^2)$, where

$$\mu_{Y_1} = \mu_{S_{a1}} - \mu_{S_1} = 14.721\,\text{ksi}$$

$$\sigma_{Y_1} = \sqrt{\sigma_{S_{a1}}^2 + \sigma_{S_1}^2} = 3.367\,\text{ksi}$$

Thus, the probability of failure due to excessive normal stress is

$$p_{f1} = \Pr(Y_1 < 0) = \Pr\left(\frac{Y_1 - \mu_{Y_1}}{\sigma_{Y_1}} < \frac{-\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi\left(\frac{-\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi(-4.3726) = 6.1384\times 10^{-6}$$ **Ans.**