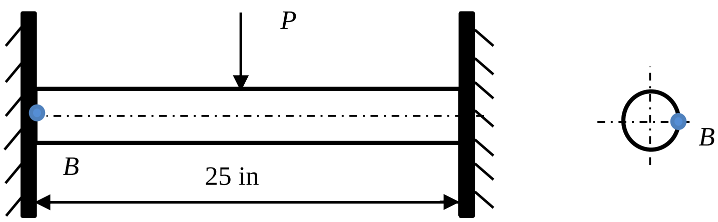
6-9. A random force $P \sim N(400, 30^2)$ lb is acting on the center of a shaft that has a radius of 0.5 in. Determine the probability of failure at point B due to excessive shear stress. The allowable shear stress follows $S_a \sim N(0.6, 0.05^2)$ ksi. Assume that P and S_a are independent.



Solution:

Section Properties

$$A = \pi(0.5)^2 = 0.7854\,\text{in}^2$$

$$I = \frac{1}{4}\pi(0.5^4) = 0.0491\,\text{in}^4$$

Normal Stress at Point B

$$\sigma_1 = 0$$

Shear Stress at Point B

$$S_1 = \frac{VQ_1}{It}$$

where

$$V = P/2, \quad t = 2r = 1$$

$$Q_1 = \overline{y}'A' = \left(\frac{4(0.5)}{3\pi}\right)\left(\frac{\pi(0.5)^2}{2}\right)$$

Thus

$$S_1 = \frac{VQ_1}{It} = \frac{(P/2)Q_1}{It} = \frac{P}{2(0.0491)(1)}\left(\frac{4(0.5)}{3\pi}\right)\left(\frac{\pi(0.5)^2}{2}\right) = (0.8488)P$$

Since $P \sim N(400, 30^2)\,\text{lb}$, we have

$$\mu_{S_1} = (0.8488)\mu_P = 0.34\,\text{ksi}$$

$$\sigma_{S_1} = (0.8488)\sigma_P = 0.03\,\text{ksi}$$

Set $Y_1 = S_a - S_1$, then $Y_1 \sim N(\mu_{Y_1}, \sigma_{Y_1}^2)$, where

$$\mu_{Y_1} = \mu_{S_a} - \mu_{S_1} = 0.26\,\text{ksi}$$

$$\sigma_{Y_1} = \sqrt{\sigma_{S_a}^2 + \sigma_{S_1}^2} = 0.06\,\text{ksi}$$

Thus, the probability of failure due to excessive normal stress is

$$p_{f1} = \Pr(Y_1 < 0) = \Pr\left(\frac{Y_1 - \mu_{Y_1}}{\sigma_{Y_1}} < \frac{-\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi\left(\frac{-\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi(-4.642) = 1.725\times 10^{-6}$$ **Ans.**