7-2. A sportsman is performing a pole vault, and the minimum radius of curvature of the pole is estimated to be 5 m. The pole has a diameter of 42 mm and its material has a Young's modulus of $E \sim N(131, 10^2)$ GPa . If the allowable bending stress of the pole is $S_a \sim N(750, 50^2)$ MPa , determine the probability of failure. Assume E and S_a are independent.

Solution:

Moment-Curvature Relationship could be calculated by

$$\frac{1}{\rho} = \frac{M}{EI}, \quad \text{in which} \quad M = \frac{I}{c}\sigma .$$

Thus, $$\frac{1}{\rho} = \frac{\frac{I}{c}\sigma}{EI} \Rightarrow \sigma = \frac{Ec}{\rho} = \frac{E\left(\frac{0.042}{2}\right)}{5} = 0.0042E$$

Set $Y = S_a - S_{\max}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{S_a} - \mu_\sigma = \mu_{S_a} - 0.0042\mu_E = 750 - 550.2 = 199.8 \text{ MPa}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + (0.0042)^2 \sigma_E^2} = \sqrt{(50)^2 + (0.0042)^2 (10^4)^2} = 65.3 \text{ MPa}$$

The probability of failure of the pole is then calculated by

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-3.0598) = 1.108 \times 10^{-3}$$ **Ans.**