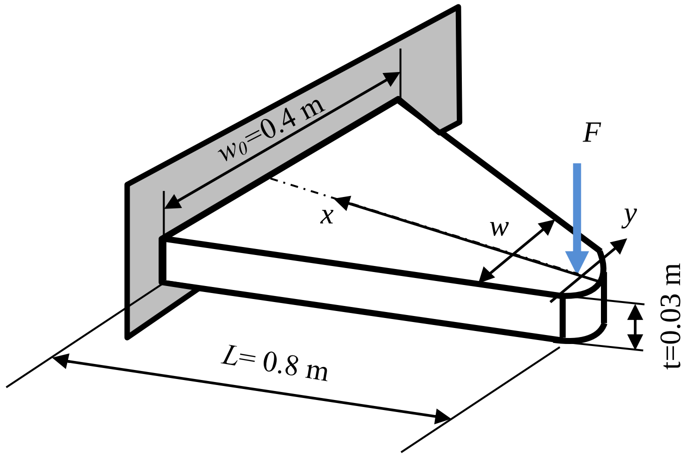
7-4. A cantilever is subject to a vertical force $F \sim N(1.4, 0.12)$ kN as shown. The dimensions of the cantilever are also shown in the figure. If the allowable normal stress of the cantilever is $S_a \sim N(28, 2^2)$ MPa, determine the probabilities of failure caused by excessive bending stress. Neglect the weight of the cantilever.



Solution:

Section properties:

$$I = \frac{1}{12}(w)(t)^3$$

The bending stress can be expressed by

$$S = \frac{Mc}{I} = \frac{(Fx)(t/2)}{wt^3/12} = \frac{Fx}{wt^2/6}$$

When $x = L$, $w = w_0$

$$S = \frac{Mc}{I} = \frac{FL}{w_0 t^2/6} = \frac{0.8F}{0.4(0.03)^2/6} = 1.33\times10^4 F$$

Set $Y = S_a - S$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{S_a} - \mu_{S_{\max}} = \mu_{S_a} - 1.33\times10^4 \mu_F = 28\times10^6 - 1.33\times10^4 \times 1.4\times10^3 = 9.38\times10^6 \text{ Pa}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + (1.33\times10^4)^2\sigma_F^2} = \sqrt{(2\times10^6)^2 + (1.33\times10^4)^2(0.12\times10^3)^2} = 2.56\times10^6 \text{ Pa}$$

The probability of failure of the cantilever caused by excessive bending stress is

$$p_f = \Pr(Y<0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-3.66) = 1.26\times10^{-4}$$ **Ans.**