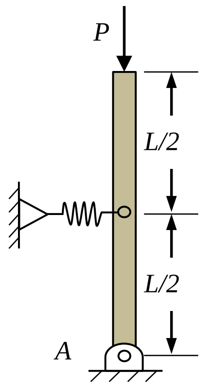
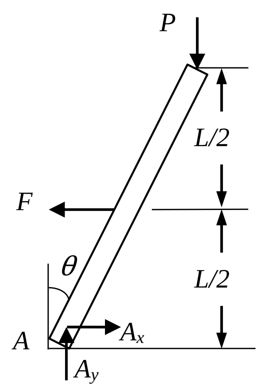
8-1. A vertical force P acts on a bar as shown in the figure. If the length of the bar L follows a normal distribution $L \sim N(1.2, 0.01^2)\,\text{m}$, determine the distribution of its critical buckling load of the bar. Assume that the spring has a stiffness of $k = 500\ \text{N/m}$.



Solution:

The free body diagram is shown below. The disturbing force F can be determined by summing moments about A.



$$\curvearrowleft +\Sigma M_A = 0, \quad F(L/2) - P(L\theta) = 0$$

Then, $F = 2P\theta$. The restoring spring force F_s can be determined by

$$F_s = k(L\theta/2) = kL\theta/2$$

For the mechanism to be on the verge of buckling, the disturbing force F must be equal to the spring force F_s. Then, we have

$$2P_{cr}\theta = kL\theta/2$$

$$P_{cr} = kL/4$$

Since $L \sim N(1.2, 0.01^2)\,\text{m}$, P_{cr} also follows normal distribution $P_{cr} \sim N(\mu_{P_{cr}}, \sigma_{P_{cr}}^2)\,\text{N}$, in which

$$\mu_{P_{cr}} = k\mu_L / 4 = 0.5\times10^3 \times 1.2 / 4 = 150 \text{ N}$$

$$\sigma_{P_{cr}} = k\sigma_L / 4 = 0.5\times10^3 \times 0.01 / 4 = 1.25 \text{ N}$$

Thus, the critical buckling load of the bar P_{cr} follows $P_{cr} \sim N(150,\ 1.25^2)$ N . **Ans.**