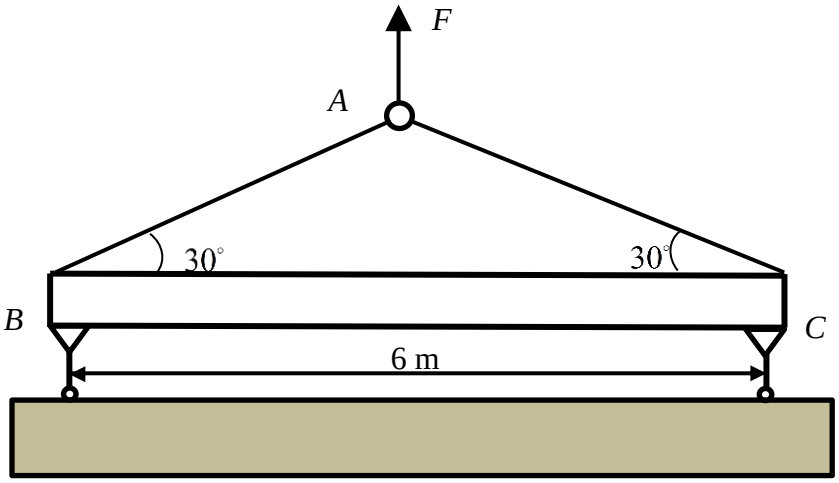
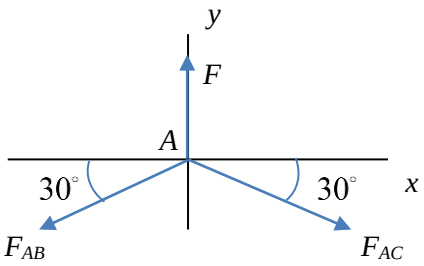
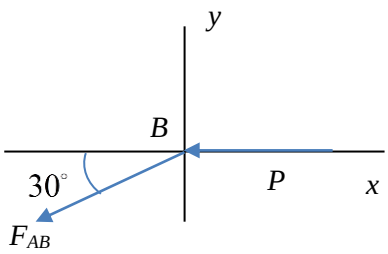
8-10. A steel rod BC with $d = 80\,\text{mm}$ is used as a strongback. A force $F \sim N\left(80, 5^2\right)\text{kN}$ is used to slowly lift the weight as shown in the figure. If the modulus of elasticity follows $E \sim N\left(200, 20^2\right)\text{GPa}$. Determine the probability of failure buckling. Assume that E and F are independent.



Weight

Solution:





Joint A :

$$\sum F_x = 0: \quad F_{AB}\cos 30^\circ - F_{Ac}\cos 30^\circ = 0, \quad F_{AB} = F_{Ac}$$

$$\sum F_y = 0: \quad F - 2F_{AB}\sin 30^\circ = 0, \quad F_{AB} = F_{Ac} = F$$

Joint B :

$$\sum F_x = 0: \qquad F_{AB}\cos 30^\circ - P = 0, \qquad P = \frac{\sqrt{3}}{2}F$$

The moment of inertia of the rod is

$$I = \frac{\pi d^4}{4} = 2.011\times 10^{-6}\ \text{m}^2$$

Thus, the buckling load is calculated by

$$F_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{(3.14)^2(2.011\times 10^{-6})}{(1\times 6)^2}E = (5.512\times 10^{-7})E, \qquad \text{where } K = 1.$$

Set $Y = F_{cr} - P$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{F_{cr}} - \mu_P = (5.512\times 10^{-7})\mu_E - \frac{\sqrt{3}}{2}\mu_F = 40.96\ \text{kN}$$

$$\sigma_Y = \sqrt{\sigma_{F_{cr}}^2 + \sigma_P^2} = \sqrt{(5.512\times 10^{-7})^2\sigma_E^2 + \left(\frac{\sqrt{3}}{2}\right)^2\sigma_F^2} = 11.84\ \text{kN}$$

Thus, the probability of failure is

$$p_f = \Pr(Y < 0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-3.4584) = 2.717\times 10^{-4} \qquad \textbf{Ans.}$$