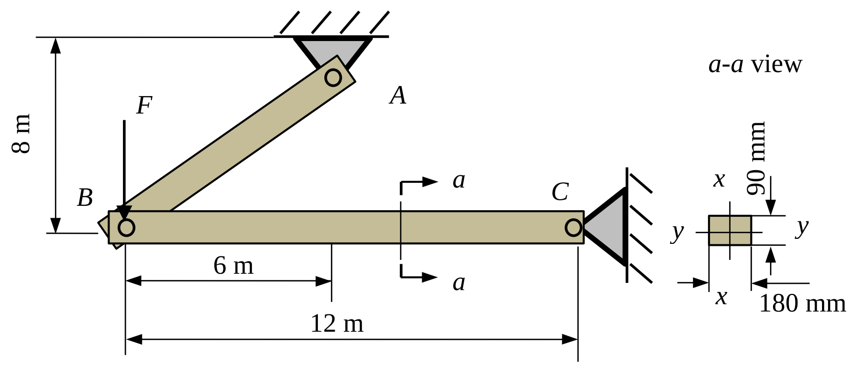
8-11. Bar *BC* is pin-connected at its ends. Load $F \sim N\left(100,\, 8^2\right)$ kN is applied to pin *B*. If the modulus of elasticity follows $E \sim N\left(200,\, 20^2\right)$ GPa. Determine the distribution of the critical buckling load about the *y*-*y* axis. Also determine the probability buckling of Bar *BC*. Assume that *E* and *F* are independent.



Solution:

Section Properties

$$I_y = \frac{1}{12}(0.18)(0.09^3) = 1.094\times10^{-5}\ \text{m}^4$$

By analyzing the equilibrium of joint *B*, we have

$$F_{AB}\left(\frac{6}{10}\right) - F_{BC} = 0$$

$$F_{AB}\left(\frac{8}{10}\right) - F = 0$$

Thus, the compressive force developed in the bar *BC* is $F_{BC} = \left(\frac{3}{4}\right)F$.

The critical buckling load about *y*-*y* axis is

$$F_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{(3.14)^2(1.094\times10^{-5})}{(1\times12)^2}E = (7.495\times10^{-7})E\,; \qquad K = 1.$$

Since $E \sim N\left(200, 20^2\right)\,\text{GPa}$, we have

$$\mu_{F_{cr}} = \left(7.495\times10^{-7}\right)\mu_E = 149.89\,\text{kN}$$

$$\sigma_{F_{cr}} = \left(7.495\times10^{-7}\right)\sigma_E = 14.99\,\text{kN}$$

Thus, the critical buckling load follows $F_{cr} \sim N\left(149.89, 14.99^2\right)\,\text{kN}$. **Ans.**

Set $Y = F_{cr} - F_{BC}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{F_{cr}} - \mu_{F_{BC}} = (7.495\times10^{-7})\mu_E - \frac{3}{4}\mu_F = 74.90\,\text{kN}$$

$$\sigma_Y = \sqrt{\sigma_{F_{cr}}^2 + \sigma_{F_{BC}}^2} = \sqrt{(7.495\times10^{-7})^2\sigma_E^2 + \left(\frac{3}{4}\right)^2\sigma_F^2} = 16.15\,\text{kN}$$

Thus, the probability of failure is

$$p_f = \Pr(Y<0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-4.6387\right) = 1.7533\times10^{-6}$$ **Ans.**