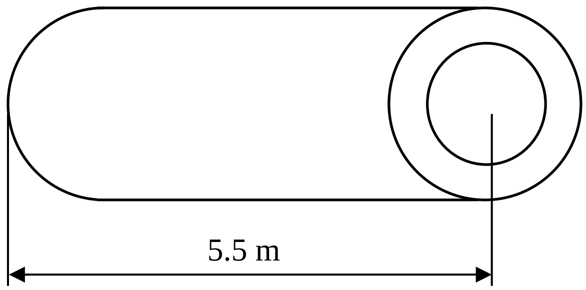
8-12. A 5.5-m long steel hollow circular tube has an outer diameter of $d_1 = 120\,\text{mm}$ and inner diameter of $d_2 = 100\,\text{mm}$. The tube is pinned at both ends, and an axial force $P \sim N\left(160,\, 20^2\right)\,\text{kN}$ is acting on it. The modulus of elasticity follows $E \sim N\left(200,\, 20^2\right)\,\text{GPa}$. Determine the distribution of the critical axial buckling load. Also, determine the probability of buckling. Assume that E and P are independent.



Solution:

Section properties

$$I = \frac{1}{64}(3.14)\left[(0.12^4) - (0.1^4)\right] = 5.27\times10^{-6}\ \text{m}^4$$

The critical axial buckling load is

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{(3.14)^2(5.27\times10^{-6})}{(1\times5.5)^2}E = (1.72\times10^{-6})E, \quad \text{where} \quad K = 1.$$

Since $E \sim N\left(200,\, 20^2\right)\,\text{GPa}$, we have

$$\mu_{P_{cr}} = \left(1.72\times10^{-6}\right)\mu_E = 343.89\,\text{kN}$$

$$\sigma_{P_{cr}} = \left(1.72\times10^{-6}\right)\sigma_E = 34.39\,\text{kN}$$

Thus, the critical axial buckling load follows $P_{cr} \sim N\left(343.89,\ 34.39^2\right)$ kN . **Ans.**

Set $Y = P_{cr} - P$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{P_{cr}} - \mu_P = 183.89 \text{ kN}$$

$$\sigma_Y = \sqrt{\sigma_{P_{cr}}^2 + \sigma_P^2} = \sqrt{\sigma_{P_{cr}}^2 + \mu_P^2} = 39.78 \text{ kN}$$

Thus, the probability of failure is

$$p_f = \Pr(Y < 0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-4.6224) = 1.8965 \times 10^{-6}$$ **Ans.**