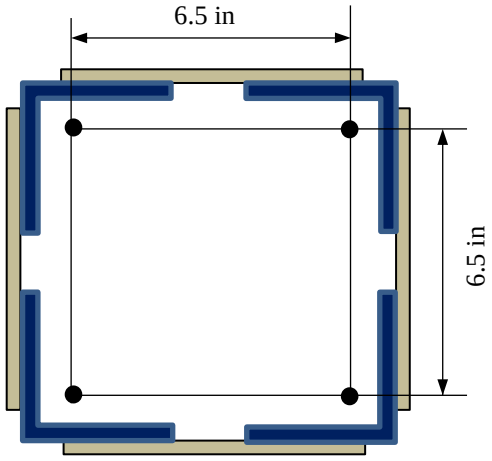
8-13. A 22-ft column consists of four steel angles laced together. The cross-section is shown in the figure. The column is pin connected at both ends. Each of these angles has an area of $A = 3.5\,\text{in}^2$, and moments of inertia of $I_x = I_y = 2.5\,\text{in}^4$. If an axial load $P \sim N\left(3.2\times10^5,\ \left(4\times10^4\right)^2\right)\text{lb}$ is applied to the column, and the modulus of elasticity follows $E \sim N\left(29\times10^6,\ \left(2\times10^6\right)^2\right)\text{psi}$, determine the distribution of the critical axial buckling load. Also, determine the probability of buckling. Assume that *E* and *P* are independent.



Solution:

Section properties of the column

$$I_{x\_col} = I_{y\_col} = 4\left(2.5 + 3.5\times(6.5/2)^2\right) = 157.88\,\text{in}^4$$

The critical axial buckling load is given by

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{(3.14)^2(157.88)}{(1\times22\times12)^2}E = (0.0224)E, \qquad \text{where } K = 1.$$

Since $E \sim N\left(29\times10^6,\ \left(2\times10^6\right)^2\right)\text{psi}$, we have

$$\mu_{P_{cr}} = (0.0224)\,\mu_E = 6.48\times10^5\ \text{lb}$$

$$\sigma_{P_{cr}} = (0.0224)\sigma_E = 4.47\times10^4 \text{ lb}$$

Thus, the critical axial buckling load follows $P_{cr} \sim N\left(6.48\times10^5,\ \left(4.47\times10^4\right)^2\right)$ lb. **Ans.**

Set $Y = P_{cr} - P$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{P_{cr}} - \mu_P = 3.28\times10^5 \text{ lb}$$

$$\sigma_Y = \sqrt{\sigma_{P_{cr}}^2 + \sigma_P^2} = \sqrt{\sigma_{P_{cr}}^2 + \mu_P^2} = 6\times10^4 \text{ lb}$$

Thus, the probability of failure is

$$p_f = \Pr(Y < 0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-5.4729) = 2.2136\times10^{-8}$$ **Ans.**