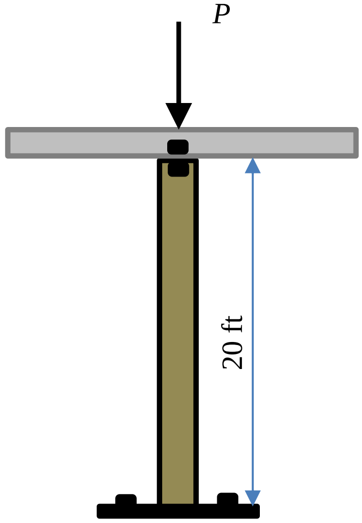
8-14. A $W14\times34$ wide flange beam is used to support a vertical force $P\sim N\left(80,\,8^2\right)$ kip as shown in the figure. The beam is 20-ft long and pinned connected at both of its ends. The modulus of elasticity follows $E\sim N\left(29\times10^6,\,\left(2\times10^6\right)^2\right)$psi . Determine the distribution of the critical buckling load. Also, determine the probability buckling. Assume that E and P are independent.



Solution:

Section roperties of the $W14\times34$ wide flange beam are

$$A=10\,\text{in}^2,\qquad I_y=23.3\,\text{in}^4$$

Then, the critical axial buckling load is

$$P_{cr}=\frac{\pi^2EI_y}{\left(KL\right)^2}=\frac{(3.14)^2(23.3)}{\left(1\times20\times12\right)^2}E=(0.004)E\,;\qquad K=1.$$

Since $E\sim N\left(29\times10^6,\,\left(2\times10^6\right)^2\right)$psi , we have

$$\mu_{P_{cr}}=\left(0.004\right)\mu_E=115.78\,\text{kip}$$

$$\sigma_{P_{cr}} = (0.004)\sigma_E = 7.98\,\text{kip}$$

Thus, the critical buckling load follows $P_{cr} \sim N\left(115.78,\ 7.98^2\right)\text{kip}$. **Ans.**

Set $Y = P_{cr} - P$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{P_{cr}} - \mu_P = 35.78\,\text{kip}$$

$$\sigma_Y = \sqrt{\sigma_{P_{cr}}^2 + \sigma_P^2} = \sqrt{\sigma_{P_{cr}}^2 + \mu_P^2} = 11.3\,\text{kip}$$

Thus, the probability of failure is

$$p_f = \Pr(Y < 0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-3.1655\right) = 7.7411 \times 10^{-4}$$ **Ans.**