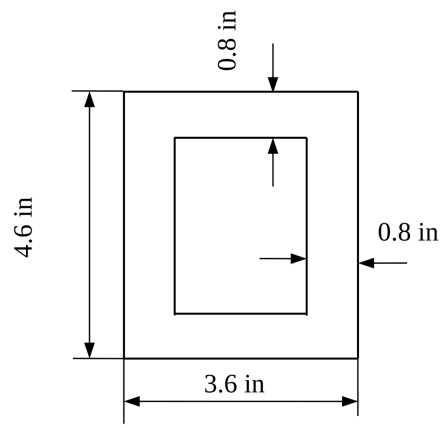
8-15. A 14-ft long tube is fixed at both ends. The cross-sectional area of this tube is shown in the figure. If the modulus of elasticity follows $E \sim N\left(29\times10^6,\ \left(2\times10^6\right)^2\right)$ psi . Determine the distribution of the critical axial buckling load. If the axial load acting on the column folllows $P \sim N\left(150,\ 15^2\right)$ kip , determine the probability of failure. Assume that $E$ and $P$ are independent and Euler's formula is available.




**Solution:**

The section property is

$$I = \frac{1}{12}(3.6)(4.6^3) - \frac{1}{12}(2)(3^3) = 24.7\ \text{in}^4$$

Then, the critical axial buckling load is

$$P_{cr} = \frac{\pi^2 EI}{\left(KL\right)^2} = \frac{(3.14)^2(24.7)}{\left(0.5\times14\times12\right)^2}E = (0.0086)E\,; \qquad K = 1.$$

Since $E \sim N\left(29\times10^6,\ \left(2\times10^6\right)^2\right)$ psi , we have

$$\mu_{P_{cr}} = \left(0.0086\right)\mu_E = 250.49\ \text{kip}$$

$$\sigma_{P_{cr}} = \left(0.0086\right)\sigma_E = 17.275\ \text{kip}$$

Thus, the critical axial buckling load follows $P_{cr} \sim N\left(250.49,\, 17.275^2\right)$ kip . **Ans.**

Set $Y = P_{cr} - P$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{P_{cr}} - \mu_P = 100.49\ \text{kip}$$

$$\sigma_Y = \sqrt{\sigma_{P_{cr}}^2 + \sigma_P^2} = \sqrt{\sigma_{P_{cr}}^2 + \mu_P^2} = 22.88\ \text{kip}$$

Thus, the probability of failure is

$$p_f = \Pr(Y < 0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-4.3923\right) = 5.6078 \times 10^{-6}$$ **Ans.**