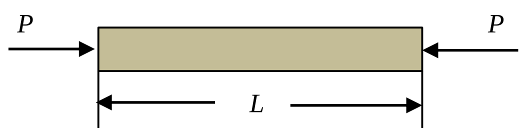
8-2. A rod is subject to two forces as shown. The rod is 16 in. long and its diameter is $d = 0.5\,\text{in}$. The yield strength of the rod is $S_y = 50\,\text{ksi}$. The modulus of elasticity follows a normal distribution $E \sim N\left(29\times10^3, (3\times10^3)^2\right)\,\text{ksi}$. The forces also follow a normal distribution $P \sim N(2, 0.2^2)\,\text{kip}$. Find the probability of failure of the rod caused by buckling. Assume that S_y and are P are independent.



Solution:

The moment of inertia of the rod is $I = \dfrac{\pi d^4}{64}$, and the effective length factor $K = 1$. Thus, the critical buckling load of the bar can be calculated by

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2}{(KL)^2}\left(\frac{\pi d^4}{64}\right)E = \frac{(3.14)^2}{(1\times16)^2}\left(\frac{3.14\times0.5^4}{64}\right)E = 1.181\times10^{-4}E$$

Set $Y = P_{cr} - P$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{P_{cr}} - \mu_P = 1.181\times10^{-4}\mu_E - \mu_P = 1.181\times10^{-4}\times29\times10^3 - 2 = 1.42\,\text{kip}$$

$$\sigma_Y = \sqrt{\sigma_{P_{cr}}^2 + \sigma_P^2} = \sqrt{(1.181\times10^{-4}\sigma_E)^2 + \sigma_P^2} = \sqrt{(1.181\times10^{-4}\times3\times10^3)^2 + (0.2)^2} = 0.41\,\text{kip}$$

Thus, the probability of failure of the rod caused by buckling could be obtained by

$$p_f = \Pr(Y<0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-3.46) = 2.7\times10^{-4}$$ **Ans.**