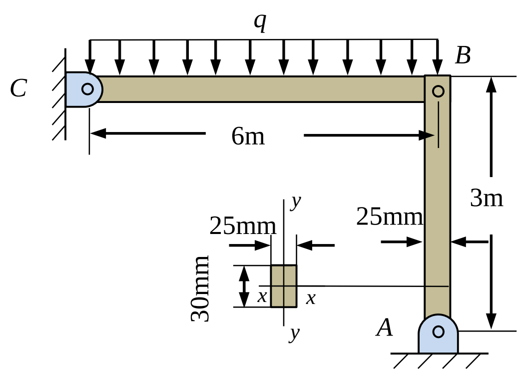
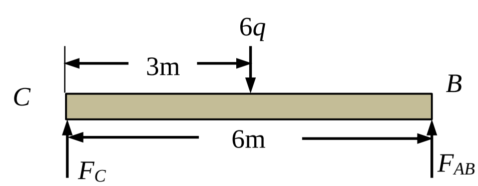
8-3. The steel bar *AB* with a rectangular cross section is pin connected at its ends. The distributed load q acting on *BC* follows a normal distribution $q \sim N(1.2,\ 0.1^2)$ kN/m, and the modulus of elasticity follows $E \sim N(200,\ 30^2)$ GPa . The yield strength of the bar follows $S_Y \sim N(120, 20^2)$ MPa . Determine the probability of falure of the bar caused by buckling. Assume that E and q are independent, and Euler's formula is valid only if the probability of failure caused by yield failure is less than 10^{-6}.



Solution:

From the free body diagram of the bar *BC*



$$\circlearrowleft + \Sigma M_C = 0, \quad 6F_{AB} - 3(6q) = 0$$

Then, we can obtain $F_{AB} = 3q$.

The effective length factor $K = 1$. The moment of inertia of the rod is

$$I = \frac{bh^3}{12} = \frac{1}{12}(0.03)(0.025)^3 = 3.9 \times 10^{-8}\ \text{m}^4$$

Thus, the critical buckling load of the bar can be calculated by

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} == \frac{(3.14)^2}{(1 \times 3)^2}(3.9 \times 10^{-8})E = 4.27 \times 10^{-8} E$$

Set $Y = P_{cr} - F_{AB}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{P_{cr}} - \mu_{F_{AB}} = 4.27\times10^{-8}\,\mu_E - 3\mu_q = 4.27\times10^{-8}\times200\times10^{9} - 3\times1.2\times10^{3} = 4940\ \text{N}$$

$$\sigma_Y = \sqrt{\sigma_{P_{cr}}^2 + \sigma_{F_{AB}}^2} = \sqrt{(4.27\times10^{-8}\sigma_E)^2 + 9\sigma_q^2} = \sqrt{(4.27\times10^{-8}\times30\times10^{9})^2 + 9(0.1\times10^{3})^2} = 1316\ \text{N}$$

Thus, the probability of failure of the rod caused by buckling could be obtained by

$$p_f = \Pr\,(Y<0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-3.75) = 8.68\times10^{-5}$$ **Ans.**

Check:

$$S_{cr} = \frac{F_{AB}}{A} = \frac{3q}{0.03\times0.025} = 4\times10^{3}q$$

Thus, $$\mu_{S_{cr}} = 4\times10^{3}\mu_q = 4\times10^{3}\times1.2\times10^{3} = 4.8\ \text{MPa}$$

$$\sigma_{S_{cr}} = 4\times10^{3}\sigma_q = 4\times10^{3}\times0.1\times10^{3} = 0.4\ \text{MPa}$$

Set $Y' = S_Y - S_{cr}$, then $Y' \sim N(\mu_{Y'}, \sigma_{Y'}^2)$, where

$$\mu_{Y'} = \mu_{S_Y} - \mu_{S_{cr}} = 120 - 4.8 = 115.2\ \text{MPa}\,,\quad \sigma_{Y'} = \sqrt{\sigma_{S_Y}^2 + \sigma_{S_{cr}}^2} = \sqrt{20^2 + 0.4^2} = 20.004\ \text{MPa}$$

$$p_f' = \Pr\,(Y'<0) = \Phi\left(\frac{-\mu_{Y'}}{\sigma_{Y'}}\right) = \Phi(-5.76) = 4.2\times10^{-9} < 10^{-6}$$ **OK.**