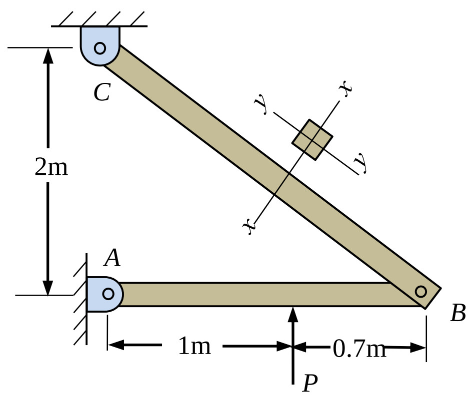
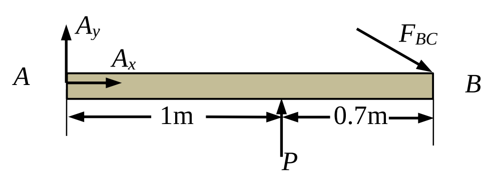
8-4. The steel bar BC has a rectangular cross section as shown in the figure. It is pin connected at its ends. The force P acting on the bar AB follows a normal distribution $P \sim N(7, 0.5^2)$ kN and the modulus of elasticity follows $E \sim N(200, 20^2)$ GPa. Determine the smallest moment of inertia I_x of the bar BC to make sure that the probability of failure of bar BC caused by x-x axis buckling is no more than 10^{-6}. Assume that E and P are independent, and $I_x < I_y$.



Solution:

From the free body diagram of the bar AB



$$\circlearrowleft + \Sigma M_A = 0, \quad P(1) - 1.7F_{BC}(2/\sqrt{2^2+1.7^2}) = 0$$

Then, we can obtain $F_{BC} = 0.772P$.

The effective length factor of x-x axis buckling is $K_x = 1$. Thus, the x-x axis critical buckling load can be calculated by

$$P_{cr\_x} = \frac{\pi^2 EI_x}{(KL)^2} == \frac{(3.14)^2}{\left(1\times\sqrt{2^2+1.7^2}\right)^2} EI_x = 7.412EI_x$$

The effective length factor of y-y axis buckling is $K_y = 0.5$. Since $I_x < I_y$, the y-y axis critical buckling load can be calculated by

$$P_{cr\_y} = \frac{\pi^2 EI_y}{(KL)^2} == \frac{(3.14)^2}{\left(0.5\times\sqrt{2^2+1.7^2}\right)^2} EI_y = 29.697 EI_y > P_{cr\_x}$$

Thus, the probability of failure of the bar *BC* caused by buckling is given by

$$P_f = \Pr\left(P_{cr\_x} < F_{BC} \cup P_{cr\_y} < F_{BC}\right) = \Pr\left(P_{cr\_x} < F_{BC}\right) = \Pr\left(P_{cr\_x} - F_{BC} < 0\right)$$

Set $Y = P_{cr\_x} - F_{BC}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{P_{cr\_x}} - \mu_{F_{BC}} = 7.412 I_x \mu_E - 0.772\mu_P = 7.412\times 2\times 10^{11} I_x - 0.772\times 7\times 10^3$$

$$\sigma_Y = \sqrt{\sigma_{P_{cr\_x}}^2 + \sigma_{F_{BC}}^2} = \sqrt{(7.412 I_x \sigma_E)^2 + (0.772\sigma_P)^2} = \sqrt{(7.412 I_x \times 2\times 10^{10})^2 + (0.772\times 0.5\times 10^3)^2}$$

Thus, P_f could be rewritten as

$$p_f = \Pr(Y<0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{7.412\times 2\times 10^{11} I_x - 0.772\times 7\times 10^3}{\sqrt{(7.412 I_x\times 2\times 10^{10})^2 + (0.772\times 0.5\times 10^3)^2}}\right) \le 10^{-6}$$

$$I_x \ge 1.49\times 10^{-8}\,\text{m}^4$$ **Ans.**