8-5. Rectangular columns are used to support a roof. The ends of a column are pin connected. The dimension of the column is shown in the figure. If the modulus of elasticity follows a normal distribution $E \sim N(1.9 \times 10^3, 150^2)$ ksi, determined the distribution of the critical load of the column. Assume that Euler's formula is valid.



Solution:

According to the cross section of the column, we can get

$$I_x = \frac{3(4)^3}{12} = 16 \text{ in}^2, \quad I_y = \frac{4(3)^3}{12} = 9 \text{ in}^2 \text{ (Controls!)}$$

Since the ends of the column are pin connected, K = 1. Thus the critical buckling load of this column can be calculated by

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{(3.14)^2 \times 9}{(1 \times 15 \times 12)^2} E = 2.74 \times 10^{-3} E$$

Thus

$$\mu_{P_{cr}} = 2.74 \times 10^{-3} \,\mu_E = 2.74 \times 10^{-3} \times 1.9 \times 10^3 = 5.21 \,\text{kip}$$
$$\sigma_{P_{cr}} = 2.74 \times 10^{-3} \,\sigma_E = 2.74 \times 10^{-3} \times 150 = 0.41 \,\text{kip}$$

The critical load of the column follows a normal distribution of $P_{cr} \sim N(5.21, 0.41^2)$ kip. Ans.