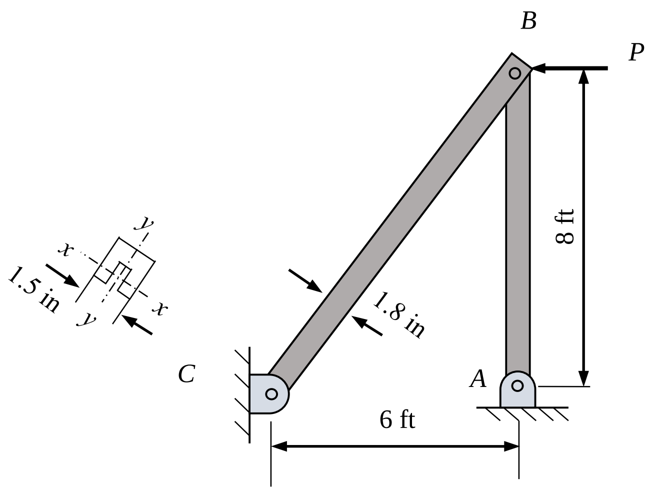
8-7. The force acting on the beam *AB* follows a normal distribution $P \sim N(5, 0.5^2)$ kip and the modulus of elasticity follows another normal distribution $E \sim N\left(29\times10^3, (2\times10^3)^2\right)$ ksi . Determine the probability of failure of beam *BC* caused by *x*-*x* axis buckling. The supports at *A*, *B* and *C* are pin connected. Assume that *E* and *P* are independent.



Solution:

From the free body diagram of beam *AB* and *BC*, we can find the load developed in beam *BC*

$$\circlearrowleft +\Sigma M_A = 0, \quad P(8) - F_{BC}(6/\sqrt{6^2+8^2})(8) = 0$$

Then, $F_{BC} = 1.67P$.

The effective length factor of *x*-*x* axis buckling is $K_x = 1$. Thus, the *x*-*x* axis critical buckling load can be calculated by

$$P_{cr\_x} = \frac{\pi^2 EI_x}{(KL)^2} == \frac{(3.14)^2\left(\dfrac{1}{12}(1.5)(1.8)^3\right)}{\left(1\times\sqrt{6^2+8^2}\times12\right)^2} E = (4.996\times10^{-4})E$$

Set $Y = P_{cr\_x} - F_{BC}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{P_{cr\_x}} - \mu_{F_{BC}} = \mu_{P_{cr\_x}} - (4.996\times10^{-4})\mu_E = 6.156 \text{ kip}$$

$$\sigma_Y = \sqrt{\sigma_{P_{cr\_x}}^2 + \sigma_{F_{BC}}^2} = \sqrt{\sigma_{P_{cr\_x}}^2 + (4.996\times10^{-4})^2 \mu_E^2} = 1.301\,\text{kip}$$

Thus, the probability of failure of the beam *BC* caused by *x*-*x* axis buckling is

$$p_f = \Pr(Y<0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-4.7315) = 1.1143\times10^{-6}$$ **Ans.**