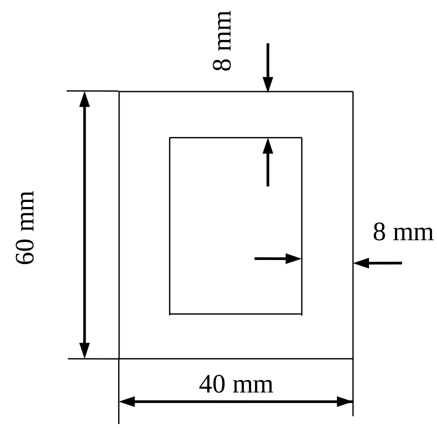
8-9. A 6-m long column is fixed at both ends. The cross-sectional area of this column is shown in the figure. If the modulus of elasticity follows $E \sim N\left(200,\, 20^2\right)\text{GPa}$. Determine the distribution of the critical axial buckling load. If the axial load acting on the column folllows $P \sim N\left(210,\, 25^2\right)\text{kN}$, determine the probability of failure. Assume that E and P are independent.



Solution:

Section Properties

$$I = \frac{1}{12}(0.04)(0.06^3) - \frac{1}{12}(0.024)(0.044^3) = 5.496 \times 10^{-7}\ \text{m}^4$$

Then, the critical axial buckling load is

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{(3.14)^2(5.496\times10^{-7})}{(0.5\times6)^2}E = (6.027\times10^{-7})E\,; \qquad K = 0.5.$$

Since $E \sim N\left(200,\, 20^2\right)\text{GPa}$, we have

$$\mu_{P_{cr}} = \left(6.027\times10^{-7}\right)\mu_E = 120.55\ \text{kN}$$

$$\sigma_{P_{cr}} = \left(6.027\times10^{-7}\right)\sigma_E = 12.06\ \text{kN}$$

Thus, the critical axial buckling load follows $P_{cr} \sim N\left(120.55, 12.06^2\right)$ kN . **Ans.**

Set $Y = P_{cr} - P$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{P_{cr}} - \mu_P = 60.55\,\text{kN}$$

$$\sigma_Y = \sqrt{\sigma_{P_{cr}}^2 + \sigma_P^2} = \sqrt{\sigma_{P_{cr}}^2 + \mu_P^2} = 13.05\,\text{kN}$$

Thus, the probability of failure is

$$p_f = \Pr(Y < 0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-4.6395\right) = 1.7465 \times 10^{-6}$$ **Ans.**