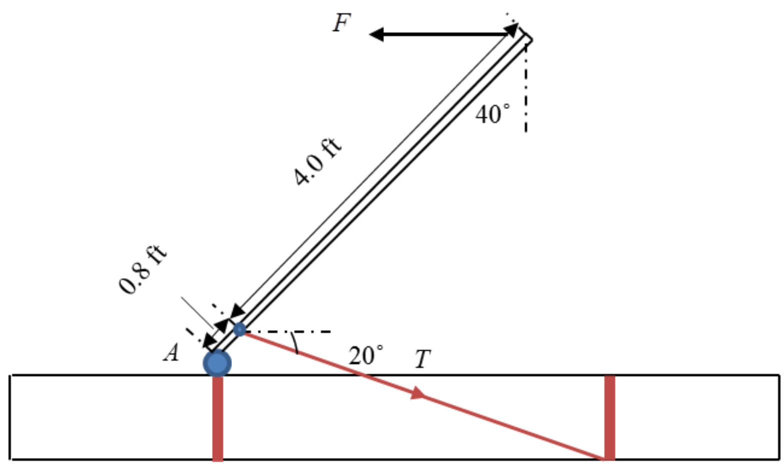
8. A horizontal force $F = 65$ lb is applied to the handle of the level in order to pull the two metal pieces B and C.

1) Determine the moment of F about A and the tension T in the cable.

2) Given $F \sim N(65, 1^2)$ lb, what is the probability that T is greater than 334 lb?

3) F is normally distributed. As a result, T is also normally distributed, and its distribution is $T \sim N(\mu_T, 4.895^2)$ lb. The strength of the cable is $T_S = 334$ lb. If the probability of failure is 5×10^{-4}, determine the distribution of F.



Solution

1)

$$M_A = FL\cos 40^\circ = 65(4.8\cos 40^\circ) = 239 \text{ lb·ft} \qquad \textbf{Ans.}$$

$$M_A = T\sin 70^\circ (0.8) \text{ g}$$

$$T = \frac{M_A}{\sin 70^\circ (0.8)} = 317.93 \text{ lb} \qquad \textbf{Ans.}$$

2)

$$T = \frac{M_A}{\sin 70^\circ (0.8)} = \frac{F(4.8)\cos 40^\circ}{\sin 70^\circ (0.8)} = \frac{6\cos 40^\circ}{\sin 70^\circ} F$$

We know $F \sim N(65, 1^2)$ lb, so we can calculate μ_T and σ_T

$$\mu_T = \frac{6\cos 40^\circ \mu_F}{\sin 70^\circ} = 317.92 \text{ lb}$$

$$\sigma_T = \frac{6\cos 40^\circ \sigma_F}{\sin 70^\circ} = 4.895 \text{ lb}$$

$$P(T > 334) = 1 - P(T \le 334) = 1 - \Phi\left(\frac{0-(\mu_T - 334)}{\sigma_T}\right) = \Phi\left(\frac{-16.08}{4.895}\right) = 0.0005$$

Ans.

3)

From the figure, we have

$$M_A = F(4.8\cos 40^\circ)$$

$$M_A = T\sin 70^\circ(0.8)$$

Thus

$$T\sin 70^\circ(0.8) = F(4.8\cos 40^\circ)$$

We know

$$P(T > 334) = 1 - P(T \le 334) = 1 - \Phi\left(\frac{0-(\mu_T - 334)}{\sigma_T}\right) = 5\times 10^{-4}$$

Thus

$$-\frac{334-\mu_T}{\sigma_T} = \Phi^{-1}(0.0005)$$

$$\mu_T = 334 + \sigma_T\Phi^{-1}(0.0005) = 317.94 \text{ lb}$$

Since we have $T\sin 70^\circ(0.8) = F(4.8\cos 40^\circ)$

$$F = \frac{T\sin 70^\circ}{6\cos 40^\circ}$$

$$\mu_F = \frac{\mu_T \sin 70^\circ}{6\cos 40^\circ} = \frac{\sin 70^\circ(317.94)}{6\cos 40^\circ} = 65 \text{ lb}$$

$$\sigma_F = \frac{\sigma_T \sin 70^\circ}{6\cos 40^\circ} = \frac{\sin 70^\circ(4.895)}{6\cos 40^\circ} = 1 \text{ lb}$$

The distribution of F is: $F \sim N(65,1^2)$ lb **Ans.**