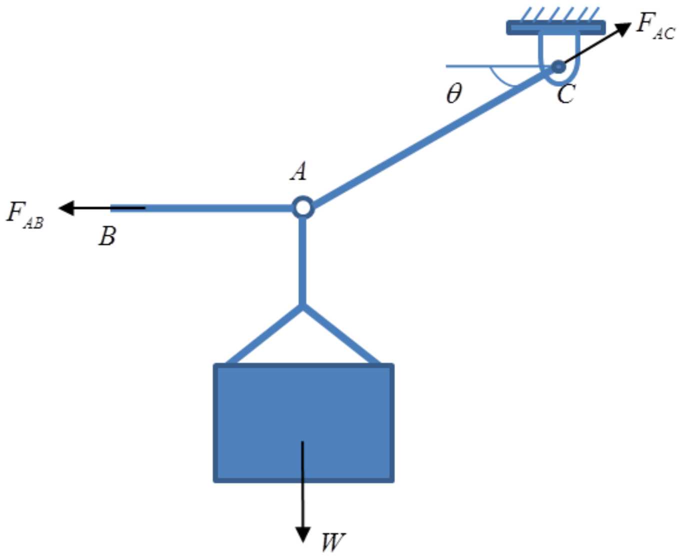
1. The weight of the crate follows a normal distribution $W \sim N(600, 20^2)$ lb, and the crate is hoisted using the ropes *AB* and *AC* with a constant speed. *AB* always remains horizontal, and θ is 12°. If the strength (maximum tension) of the ropes also follows a normal distribution $S \sim N(3000, 100^2)$ lb and *S* is independent of *W*, determine the probability that rope *AB* and *AC* will break, respectively.



Solution

$$\sum F_y = 0;\ F_{AC}\sin\theta - W = 0$$
$$\sum F_x = 0;\ F_{AC}\cos\theta - F_{AB} = 0$$

For $W \sim N(600, 20^2)$ lb and $\theta = 12^\circ$

$$F_{AC} = \frac{W}{\sin\theta}$$

$$\mu_{F_{AC}} = \frac{\mu_W}{\sin\theta} = \frac{600}{\sin 12^\circ} = 2885.8 \text{ lb}$$
$$\sigma_{F_{AC}} = \frac{\sigma_W}{\sin\theta} = \frac{20}{\sin 12^\circ} = 96.2 \text{ lb}$$
$$F_{AB} = F_{AC}\cos\theta$$
$$\mu_{F_{AB}} = \mu_{F_{AC}}\cos\theta = 2885.8(\cos 12^\circ) = 2822.7 \text{ lb}$$
$$\sigma_{F_{AB}} = \sigma_{F_{AC}}\cos\theta = 96.2(\cos 12^\circ) = 94.1 \text{ lb}$$

So the distributions of F_{AC} and F_{AB} are $F_{AC} \sim N(2885.8, 96.2^2)$ lb and $F_{AB} \sim N(2822.7, 94.1^2)$ lb, respectively.

We know the maximum tension of each rope is $S \sim N(3000,100^2)$ lb before it breaks, suppose

$$Y = S - F_{AC}$$
$$Z = S - F_{AB}$$

Thus

$$\mu_Y = \mu_S - \mu_{F_{AC}} = 3000 - 2885.8 = 114.2 \text{ lb}$$
$$\sigma_Y = \sqrt{\sigma_S{}^2 + \sigma^2_{F_{AC}}} = \sqrt{100^2 + 96.2^2} = 138.8 \text{ lb}$$
$$\mu_Z = \mu_S - \mu_{F_{AB}} = 3000 - 2822.7 = 177.3 \text{ lb}$$
$$\sigma_Z = \sqrt{\sigma_s^2 + \sigma^2_{F_{AB}}} = \sqrt{100^2 + 94.1^2} = 137.3 \text{ lb}$$

So the distributions of Y and Z are $Y \sim N(114.2, 138.8^2)$ lb and $Z \sim N(177.3, 137.3^2)$ lb, respectively.

The probability of the break of rope *AC* is $P(Y < 0)$ and the probability of the break of rope *AB* is $P(Z < 0)$,

$$P(Y < 0) = \Phi\left(\frac{0 - \mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-114.2}{138.8}\right) = 0.205 \qquad \textbf{Ans.}$$
$$P(Z < 0) = \Phi\left(\frac{0 - \mu_Z}{\sigma_Z}\right) = \Phi\left(\frac{-177.3}{137.3}\right) = 0.098 \qquad \textbf{Ans.}$$

Thus, we conclude that the probability of the break of rope *AC* is 0.205 and the probability of the break of rope *AB* is 0.098.