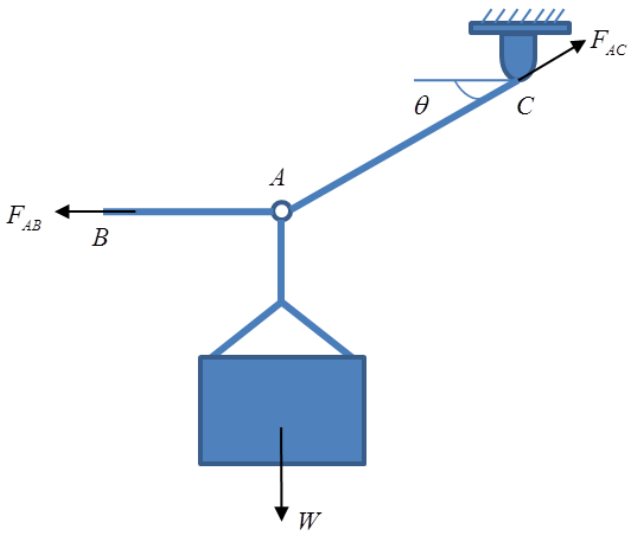
10. The weight of the crate follows a normal distribution $W \sim N(700, 10^2)$ lb and the crate is hoisted using ropes *AB* and *AC*. Each rope can withstand a maximum tension $T_{\max} \sim N(4100, 20^2)$ lb before it breaks. If *AC* always remains horizontal and θ is 10°, determine the probability that rope *AB* and *AC* will break. Note all the forces W, $T_{\max}$, F_{AB}, and F_{AC} are independently distributed.



Solution

$$\sum F_y = 0;\ F_{AC}\sin\theta - W = 0$$
$$\sum F_x = 0;\ F_{AC}\cos\theta - F_{AB} = 0$$

For $W \sim N(700, 10^2)$ lb and $\theta = 10^\circ$

$$F_{AC} = \frac{W}{\sin\theta}$$

$$\mu_{F_{AC}} = \frac{\mu_W}{\sin\theta} = 4031.1 \text{ lb}$$
$$\sigma_{F_{AC}} = \frac{\sigma_W}{\sin\theta} = 57.6 \text{ lb}$$
$$F_{AB} = F_{AC}\cos\theta$$
$$\mu_{F_{AB}} = \mu_{F_{AC}}\cos\theta = 3970 \text{ lb}$$
$$\sigma_{F_{AB}} = \sigma_{F_{AC}}\cos\theta = 56.7 \text{ lb}$$

So the distributions of F_{AC} and F_{AB} are $F_{AC} \sim N(4031.1, 57.6^2)$ lb and $F_{AB} \sim N(3970, 56.7^2)$ lb, respectively.

We know the maximum tension of each rope is $T_{\max} \sim N(4100, 20^2)$ lb before it breaks, suppose

$$Y = T_{\max} - F_{AC}$$

$$Z = T_{\max} - F_{AB}$$

Thus

$$\mu_Y = \mu_{T_{\max}} - \mu_{F_{AC}} = 68.9 \text{ lb}$$
$$\sigma_Y = \sqrt{\sigma_{T_{\max}}{}^2 + \sigma_{F_{AC}}{}^2} = 61 \text{ lb}$$
$$\mu_Z = \mu_{T_{\max}} - \mu_{F_{AB}} = 130.1 \text{ lb}$$
$$\sigma_Z = \sqrt{\sigma_{T_{\max}}{}^2 + \sigma_{F_{AB}}{}^2} = 60.1 \text{ lb}$$

So the distributions of Y and Z are $Y \sim N(68.9, 61^2)$ lb and $Z \sim N(130.1, 60.1^2)$ lb, respectively.

The probability of the break of rope *AC* is $P(Y < 0)$ and the probability of the break of rope *AB* is $P(Z < 0)$,

$$P(Y < 0) = \Phi\left(\frac{0 - \mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-68.9}{61}\right) = 0.129$$ **Ans.**

$$P(Z < 0) = \Phi\left(\frac{0 - \mu_Z}{\sigma_Z}\right) = \Phi\left(\frac{-130.1}{60.1}\right) = 0.015$$ **Ans.**

Thus, we conclude that the probability of the break of rope *AC* is 0.129 and the probability of the break of rope *AB* is 0.015.