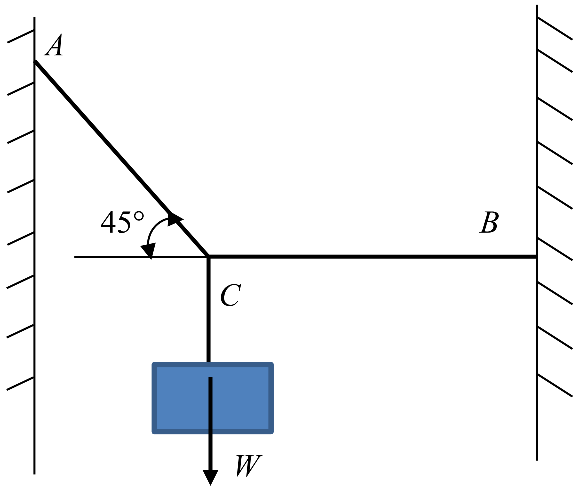
11. The weight follows a normal distribution $W \sim N(500, 5^2)$ N. The resistances of cable AC and cable AB are also normally distributed with $S_{AC} \sim N(720, 6^2)$ N and $S_{BC} \sim N(520, 8^2)$ N, respectively. Determine which cable is more likely to break. W, S_{AC}, and S_{BC} are independent.



Solution

$$\Sigma F_x = 0;\ T_{BC} - T_{AC}\cos 45^\circ = 0$$

$$\Sigma F_y = 0;\ T_{AC}\sin 45^\circ - W = 0$$

For cable AC

$$T_{AC} = \frac{W}{\sin 45^\circ}$$

$$\mu_{T_{AC}} = \frac{\mu_W}{\sin 45^\circ} = 707 \text{ N}$$

$$\sigma_{T_{AC}} = \frac{\sigma_W}{\sin 45^\circ} = 7.07 \text{ N}$$

We construct the function

$$Y = S_{AC} - T_{AC}$$

$$\mu_Y = \mu_{S_{AC}} - \mu_{T_{AC}} = 12.89 \text{ N}$$

$$\sigma_Y = \sqrt{\sigma_{S_{AC}}^2 + \sigma_{T_{AC}}^2} = 9.27 \text{ N}$$

The probability that cable AC will break is

$$p_{f\_AC} = \Pr(Y < 0) = \Phi(-\mu_Y / \sigma_Y) = 0.0822$$

For cable *BC*

$$T_{BC} = T_{AC} \cos 45^\circ = W$$

$$\mu_{T_{BC}} = \mu_W = 500 \text{ N}$$

$$\sigma_{T_{BC}} = \sigma_W = 5 \text{ N}$$

We construct the function

$$Z = S_{BC} - T_{BC}$$

$$\mu_Z = \mu_{S_{BC}} - \mu_{T_{BC}} = 20 \text{ N}$$

$$\sigma_Z = \sqrt{\sigma_{S_{BC}}^2 + \sigma_{T_{BC}}^2} = 9.43 \text{ N}$$

The probability that cable *BC* will break is

$$p_{f\_BC} = \Pr(Z < 0) = \Phi(-\mu_Z / \sigma_Z) = 0.017$$

We can see

$$p_{f\_BC} < p_{f\_AC}$$

Thus, cable *AC* is more likely to break. **Ans.**