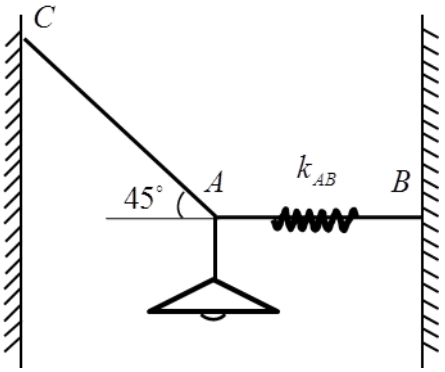
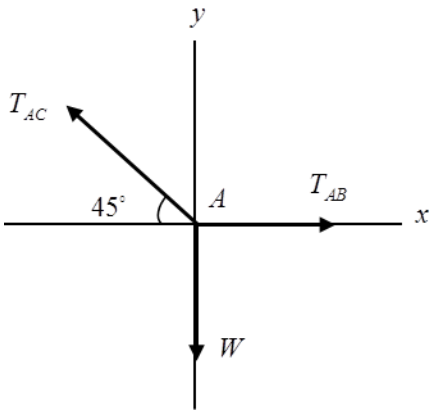
3. The unstretched length of spring AB follows a normal distribution $l_{AB} \sim N(0.5, 0.01^2)$ m and the weight of the lamp also follows a normal distribution $W \sim N(100, 2^2)$ N. The spring has a stiffness of $k_{AB} = 350$ N/m. Determine the distribution of the length of cord AB shown in the figure. l_{AB} and W are independently distributed.



Solution



$$\Sigma F_x = 0;\ T_{AB} - T_{AC}\cos 45^\circ = 0$$

$$\Sigma F_y = 0;\ T_{AC}\sin 45^\circ - W = 0$$

From the above two equations, we have

$$T_{AB} = \frac{W\cos 45^\circ}{\sin 45^\circ} = W$$

The distribution of the stretch of spring AB is therefore

$$T_{AB} = k_{AB} s_{AB}$$

$$s_{AB} = \frac{T_{AB}}{k_{AB}} = \frac{W}{k_{AB}}$$

$$\mu_{s_{AB}} = \frac{\mu_W}{k_{AB}} = \frac{100}{350} = 285.7 \text{ mm}$$

$$\sigma_{s_{AB}} = \frac{\sigma_W}{k_{AB}} = \frac{2}{350} = 5.7 \text{ mm}$$

$$s_{AB} \sim N(285.7, 5.7^2) \text{ mm}$$

So the stretch length is

$$L_{AB} = l_{AB} + s_{AB}$$

$$\mu_{L_{AB}} = \mu_{l_{AB}} + \mu_{s_{AB}} = 0.5 + 0.2857 = 0.79 \text{ m}$$

$$\sigma_{L_{AB}} = \sqrt{\sigma_{l_{AB}}^2 + \sigma_{s_{AB}}^2} = \sqrt{0.01^2 + 0.0057^2} = 0.01 \text{ m}$$

Thus, the stretch length of cord *AB* is $L_{AB} \sim N(0.79, 0.01^2)$ m. **Ans.**