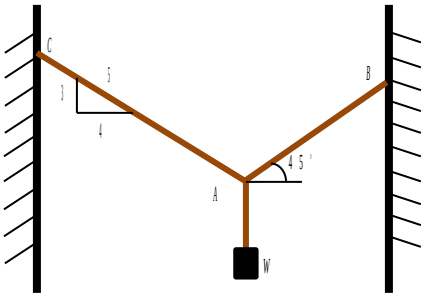
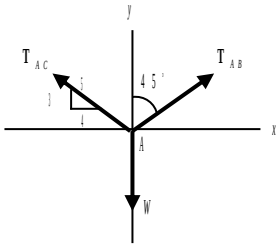
4. If the load has a weight $W \sim N(25, 2^2)$ lb and the system is in the equilibrium position shown, what are the probabilities of failure if the allowable tensions of *AB* is 25 lb and *AC* is 22 lb in the cables, respectively?



Solution

There are three forces acting on *A*, as shown in the figure below.



So we have

$$\Sigma F_x = 0;\ T_{AB}\cos 45^\circ - T_{AC}(\frac{4}{5}) = 0$$

$$\Sigma F_y = 0;\ T_{AB}\sin 45^\circ + T_{AC}(\frac{3}{5}) - W = 0$$

From above equations, $T_{AB} = \frac{4\sqrt{2}}{5}T_{AC}$, $T_{AC} = \frac{5}{7}W$. With $W \sim N(25, 2^2)$ lb, we have

$$T_{AB} \sim N(20.20, 1.62^2) \text{ lb}$$

$$T_{AC} \sim N(17.86, 1.43^2) \text{ lb}$$

For cable *AB*, the probability of failure is

$$P(Y \geq 25) = 1 - P(Y < 25) = 1 - \Phi\left(\frac{25 - 20.20}{1.62}\right) = 0.0015$$ **Ans.**

For cable *AC*, the probability of failure is

$$P(Z \geq 22) = 1 - P(Y < 22) = 1 - \Phi\left(\frac{22 - 17.86}{1.43}\right) = 0.0019$$ **Ans.**