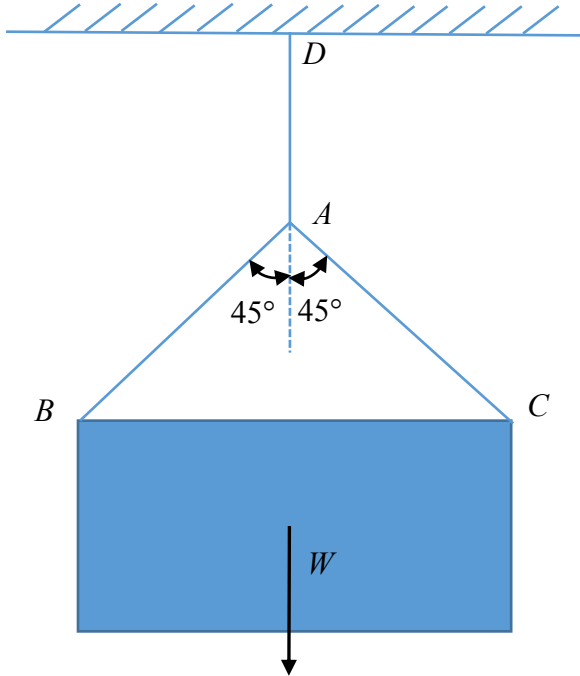


5. Cable BAC is used to lift the normally distributed load $W \sim N(100, 5^2)$ lb. If the system is in the position shown below, determine the probability that the system may fail. The allowable tension of the cable AB follows another normal distribution $T \sim N(80, 5^2)$ N. Assume that T and W are independently distributed.



Solution

$$\sum F_y = 0; \quad T_{AB} = \frac{W}{\sqrt{2}}$$

Then, we have

$$\mu_{T_{AB}} = \frac{\mu_W}{\sqrt{2}} = 70.71 \text{ lb}$$

$$\sigma_{T_{AB}} = \frac{\sigma_W}{\sqrt{2}} = 3.536$$

Therefore, we have

$$T_{AB} \sim N(70.71, 3.536^2) \text{ lb}$$

Thus, the probability of failure of the system is that $P(Y > 0)$, where $Y = T_{AB} - T$.

Also, we have

$$\mu_Y = \mu_{T_{AB}} - \mu_T = -9.29 \text{ lb}$$

$$\sigma_Y = \sqrt{\sigma_{T_{AB}}^2 + \sigma_T^2} = 6.124$$

Finally, the probability of failure is

$$\Pr = P(Y > 0) = 1 - P(Y \leq 0) = 1 - \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = 6.46\%$$

Ans.