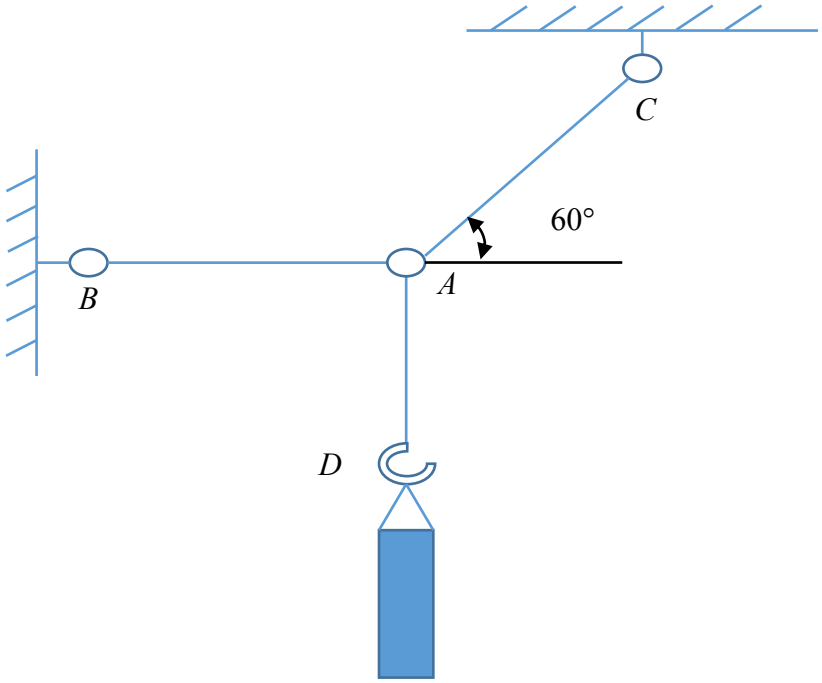
6. The load $W \sim N(10, 0.5^2)$ kg pipe is supported at *D* by a system of three cords. Determine the force in each cord. If the allowable tension of *AC* follows normal distribution $T \sim N(150, 10^2)$ N, which is independent from *W*, determine the probability of failure of cable *AC*.



Solution

$$\sum F_y = 0; \quad T_{AC} \sin 60^\circ = W$$
$$\sum F_x = 0; \quad T_{AC} \cos 60^\circ = T_{AB}$$

Thus, we could obtain

$$\mu_{T_{AC}} = \frac{\mu_W}{\sin 60^\circ} = 113.28 \text{ N}$$

$$\sigma_{T_{AC}} = \frac{\sigma_W}{\sin 60^\circ} = 5.66$$

$$\mu_{T_{AB}} = \mu_{T_{AC}} \cos 60^\circ = 56.64 \text{ N}$$

$$\sigma_{T_{AB}} = \sigma_{T_{AC}} \cos 60^\circ = 2.84$$

Thus, we have

$$T_{AB} \sim N(56.64, 2.83^2)\ \text{N and}\ T_{AC} \sim N(113.28, 5.66^2)\ \text{N}$$

Thus, the probability of failure of the system is that $P(Y > 0)$, where $Y = T_{AC} - T$.

Also, we have

$$\mu_Y = \mu_{T_{AC}} - \mu_T = -36.72\ \text{N}$$

$$\sigma_Y = \sqrt{\sigma_{T_{AC}} + \sigma_T} = 11.49$$

Finally, the probability of failure is

$$\text{Pr} = P(Y > 0) = 1 - P(Y \le 0) = 1 - \Phi(\frac{-\mu_Y}{\sigma_Y}) = 6.981 \times 10^{-4}$$ **Ans**.