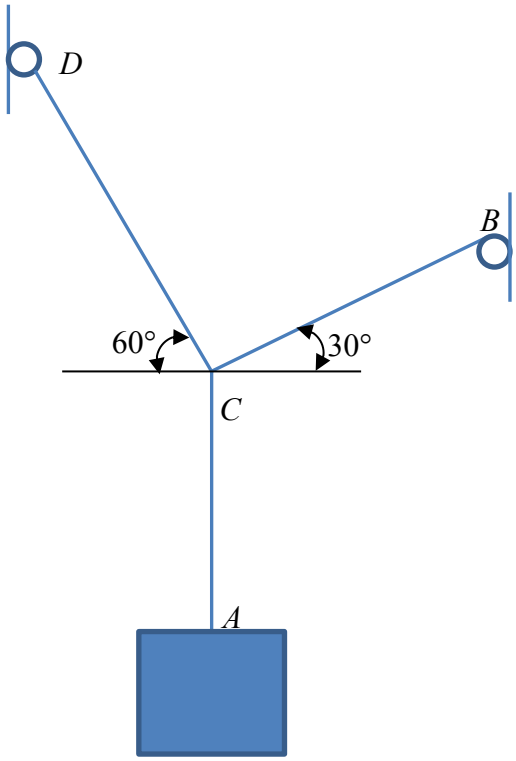
7. The cord *CD* can support a maximum load of $T \sim N(80, 2^2)$ lb. Determine the probability of failure of the system if the weight of the crate follows another independent normal distribution $W \sim N(80, 5^2)$ lb.



Solution

$$\sum F_x = 0;\ F_{CD}\cos 60^\circ = F_{BC}\cos 30^\circ$$

$$\sum F_y = 0;\ F_{CD}\sin 60^\circ + F_{BC}\sin 30^\circ = W$$

From above equations, we have

$$F_{BC} = \frac{1}{2}W \quad \text{and} \quad F_{CD} = \frac{\sqrt{3}}{2}W$$

Given $W \sim N(80, 5^2)$ lb,

$$\mu_{F_{BC}} = \frac{1}{2}\mu_W = 40 \text{ lb};\ \sigma_{F_{BC}} = \frac{1}{2}\sigma_W = 2.5 \text{ lb}$$

$$\mu_{F_{CD}} = \frac{\sqrt{3}}{2}\mu_W = 69.28 \text{ lb};\ \sigma_{F_{BC}} = \frac{\sqrt{3}}{2}\sigma_W = 4.33 \text{ lb}$$

Therefore, $F_{BC} \sim N(40, 2.5^2)$ lb, $F_{CD} \sim N(69.28, 4.33^2)$ lb

Since $F_{CD} > F_{BC}$, we use F_{CD}.

Thus, the probability of failure of the system is that $P(Y > 0)$, where $Y = F_{CD} - T$.

Also, we have

$$\mu_Y = \mu_{F_{CD}} - \mu_T = -10.72 \text{ N}$$

$$\sigma_Y = \sqrt{\sigma_{F_{CD}} + \sigma_T} = 4.77$$

Finally, the probability of failure is

$$\Pr = P(Y > 0) = 1 - P(Y \le 0) = 1 - \Phi(\frac{-\mu_Y}{\sigma_Y}) = 1.23\%$$ **Ans**.