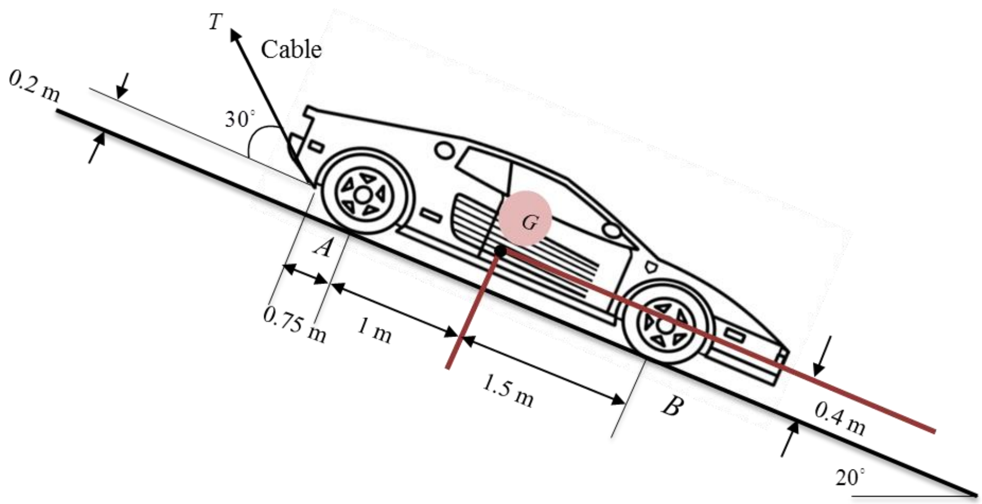
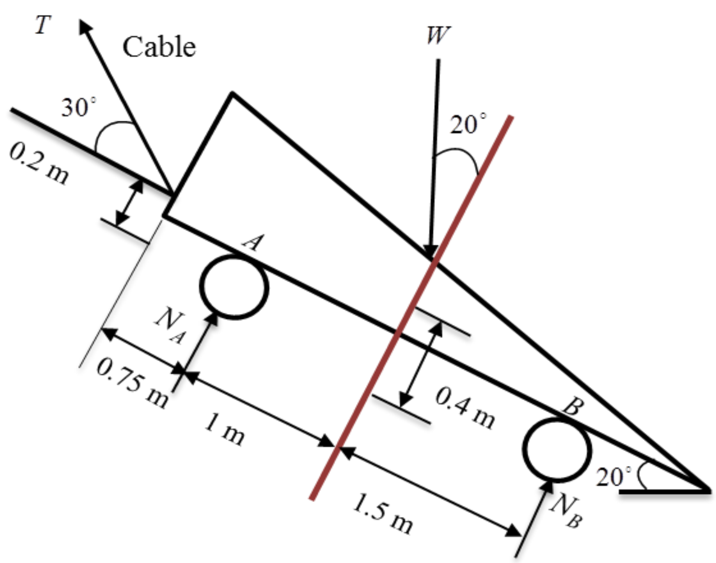
1. . 1) A car of 4 Mg is on the hill as shown. What is the tension T in the cable required to keep the car from falling down the hill?

2) The weight is given by $W \sim N(39.24, 2^2)$ kN, and the strength of the cable follows $S \sim N(19, 0.8^2)$ kN. If W and S are independent, what is the probability of failure of the cable?



Solution



1)

$$\Sigma F_x = 0$$

$$T\cos 30^\circ - 4000(9.81)\sin 20^\circ = 0$$

$$T = 15.5 \text{ kN}$$ **Ans.**

2)

$$\Sigma F_x = 0$$

$$T\cos 30^\circ - W\sin 20^\circ = 0$$

We have

$$T = \frac{W\sin 20^\circ}{\cos 30^\circ}$$

$$\mu_T = \frac{\mu_W \sin 20^\circ}{\cos 30^\circ} = 15.50 \text{ kN}$$

$$\sigma_T = \frac{\sigma_W \sin 20^\circ}{\cos 30^\circ} = 0.79 \text{ kN}$$

$$T \sim N(15.50, 0.79) \text{ kN}$$

If the stress *F* is greater than strength *S*, the cable will fail. So we define

$$Y = S - T$$

$$\mu_Y = \mu_s - \mu_T = 19 - 15.50 = 3.5 \text{ kN}$$

$$\sigma_Y = \sqrt{\sigma_S^2 + \sigma_T^2} = \sqrt{0.8^2 + 0.79^2} = 1.124 \text{ kN}$$

Then, the probability of failure is

$$P(T > S) = P(Y < 0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-3.5}{1.124}\right) = 0.0009$$ **Ans.**