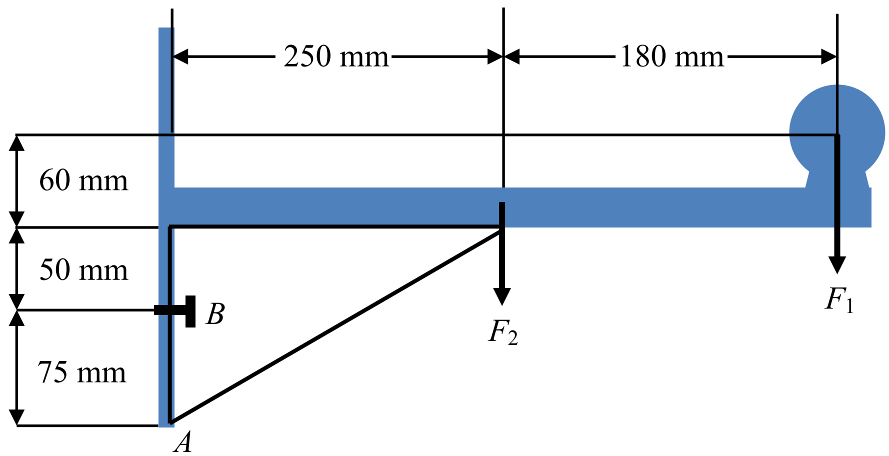
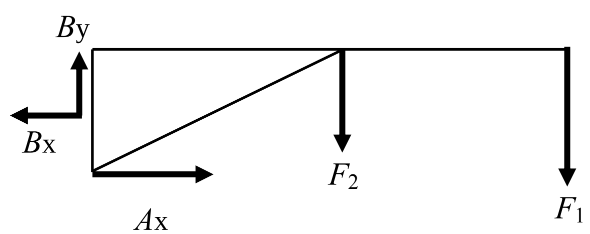
10. A bolt B holds the beam up and the bracket bear against the smooth wall at A. If the allowable horizontal and vertical loads of the bolt B are 1200 N and 220 N respectively, what are the probabilities that the actual horizontal and vertical forces of the bolt exceed their allowable forces? The beam supports two vertical forces $F_1 \sim N(16, 1.0^2)$ kg and $F_2 \sim N(5, 0.125^2)$ kg. F_1 and F_2 are independent.



Solution



$$\sum M_A = 0; \quad B_x(75) - F_2(250) - F_1(430) = 0,$$
$$\sum F_x = 0; \quad A_x = B_x,$$
$$\sum F_y = 0; \quad B_y = F_2 + F_1.$$

Since, F_1 and F_2 follow the normal distributions that are independent, we could obtain

$$\mu_{B_x} = \frac{430}{75}(9.81)\mu_{F_1} + \frac{250}{75}(9.81)\mu_{F_2} = 1063.4 \text{ N},$$

$$\sigma_{B_x} = \sqrt{(\frac{430}{75}(9.81)\sigma_{F_1})^2 + (\frac{250}{75}(9.81)\sigma_{F_2})^2} = 56.4,$$

$$\mu_{A_x} = \mu_{B_x} = 1063.4 \text{ N},$$

$$\sigma_{A_x} = \sigma_{B_x} = 56.4,$$

$$\mu_{B_y} = (9.81)(\mu_{F_1} + \mu_{F_2}) = 206 \text{ N},$$

$$\sigma_{B_y} = (9.81)\sqrt{\sigma^2{}_{F_1} + \sigma^2{}_{F_2}} = 9.9.$$

Therefore, we have the forces distributions as $A_x \sim N(1063.4, 56.4^2)$ N, $B_x \sim N(1063.4, 56.4^2)$ N and $B_y \sim N(206, 9.9^2)$ N. **Ans.**

The probability of bolt to fail in the horizontal direction is

$$P_h\left(B_x \geq 1200 \text{ N}\right) = 1 - P_h\left(B_x < 1200 \text{ N}\right) = 1 - \Phi(\frac{1200 - 1063.4}{56.4}) = 0.0077.$$ **Ans.**

The probability of bolt to fail in the vertical direction is

$$P_V\left(B_y \geq 220 \text{ N}\right) = 1 - P_V\left(B_y < 220 \text{ N}\right) = 1 - \Phi(\frac{220 - 206}{9.9}) = 0.0785.$$ **Ans.**