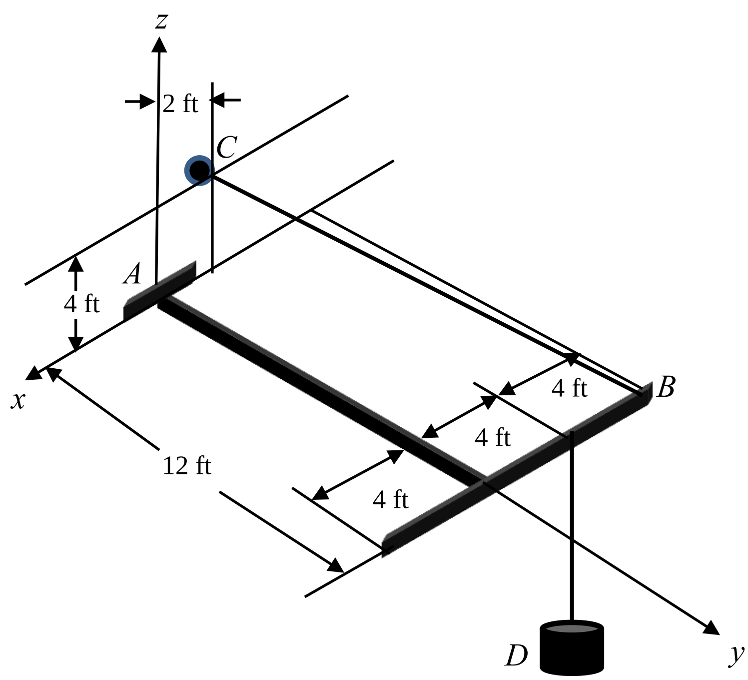
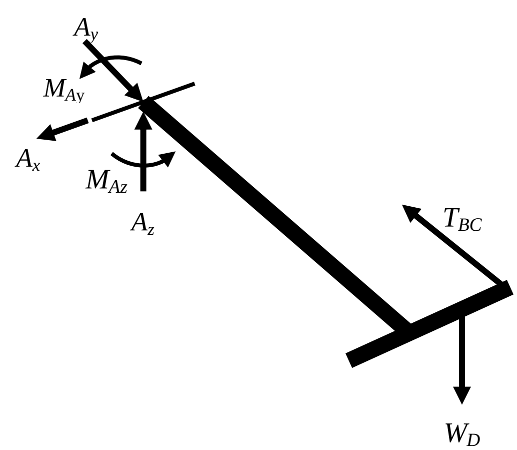
11. Component *AB* is supported by pin at *A* and cable *BC*. If the load at *D* follows a normal distribution of $N(350,5^2)$ lb, determine the distributions of *x*, *y*, *z* components of reaction at these support. And if the maximum tension of the cable is 1250 lb, what is the probability for the system to fail?



Solution



$$\mathrm{T}_{BC} = T_{BC}\{\frac{3}{7}\mathrm{i} - \frac{6}{7}\mathrm{j} + \frac{2}{7}\mathrm{k}\}\ \mathrm{ft},$$

$$\sum F_x = 0; \quad A_x + (\frac{3}{7})T_{BC} = 0,$$

$$\sum F_y = 0; \quad A_y - (\frac{6}{7})T_{BC} = 0,$$

$$\sum F_z = 0; \quad A_z - W_D + (\frac{2}{7})T_{BC} = 0,$$

$$\sum M_x = 0; \quad -W_D(12) + (\frac{2}{7})T_{BC}(12) = 0,$$

$$\sum M_y = 0; \quad M_{A_y} - W_D(4) + (\frac{2}{7})T_{BC}(8) = 0,$$

$$\sum M_z = 0; \quad M_{A_z} - (\frac{3}{7})T_{BC}(12) + (\frac{6}{7})T_{BC}(8) = 0.$$

Since, the weight of load D follows the normal distribution $N(350, 5^2)$ lb, then we could obtain

$$\mu_{T_{BC}} = \frac{7}{2}\mu_D = 1225\ \mathrm{lb},$$
$$\sigma_{T_{BC}} = \frac{7}{2}\sigma_D = 17.5,$$
$$\mu_{A_x} = -\frac{3}{2}\mu_D = -525\ \mathrm{lb},$$
$$\sigma_{A_x} = \frac{3}{2}\sigma_D = 7.5,$$
$$\mu_{A_y} = 3\mu_D = 1050\ \mathrm{lb},$$
$$\sigma_{A_y} = 3\sigma_D = 15,$$
$$A_z = 0,$$
$$\mu_{M_{A_y}} = -2\mu_D = -700\ \mathrm{lb}\cdot\mathrm{ft},$$
$$\sigma_{M_{A_y}} = 2\sigma_D = 10,$$
$$\mu_{M_{A_z}} = -3\mu_D = -1050\ \mathrm{lb}\cdot\mathrm{ft},$$
$$\sigma_{M_{A_z}} = 3\sigma_D = 15.$$

Thus, we have obtained all the distributions of x, y, z components at support A and tension on cable BC. We have $T_{BC} \sim N(1225, 17.5^2)$ lb, $A_x \sim N(-525, 7.5^2)$ lb, $A_y \sim N(1050, 15^2)$ lb, A_z=0, $M_{Ay} \sim N(-700, 10^2)$ lb·f and $M_{Az} \sim N(-1050, 15^2)$ lb·ft. **Ans.**

The maximum tension of the cable BC is 1250 lb, then the probability that the system may fail is

$$P(T_{BC} \geq 1250\ \mathrm{lb}) = 1 - P(T_{BC} < 1250\ \mathrm{lb}) = 1 - \Phi(\frac{1250 - 1225}{17.5}) = 0.0766.$$ **Ans.**